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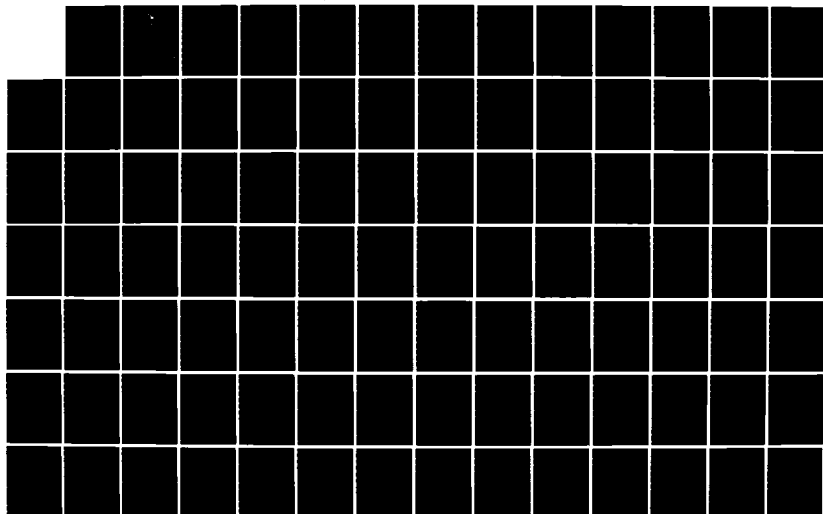
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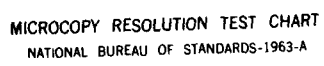
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STUDY OF TOTAL GAMMA SPECTRA CORRELATION  
FOR EXTENDING IDENTIFICATION RANGE OVER  
PHOTOPEAK ANALYSIS

THESIS

Alan W. Dooley  
Second Lieutenant, USAF

AFIT/GNE/PH /84M-2

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AUG 21 1984

DEPARTMENT OF THE AIR FORCE

AIR UNIVERSITY (ATC)

**AIR FORCE INSTITUTE OF TECHNOLOGY**

Wright-Patterson Air Force Base, Ohio

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## Preface

It has been my purpose in this thesis to study the possibility of finding a method for extending spectrum identification beyond the limitations of photopeak analysis.

To carry out this theoretical study the Monte Carlo Transport Code, Morse, was used to generate output spectra. The code is extremely flexible and has been adapted for Wright-Patterson's CDC-6600 computer.

I cannot express my gratitude enough to the personnel of RSIC at Oak Ridge National Laboratory. Their support in sending me Morse and important information was beyond the call of duty. My personal thanks to Dr. Jabo Sae Tang and Dr. M. B. Emmett for their many hours of getting me through the troubled days when I could not get Morse running.

A little closer to home, my appreciation to Dr. George John and Dr. Don Shankland for their ready answers to my detector and correlation theory questions.

Last, I wish to thank Major McKee for proposing this topic and serving as my Thesis advisor. And I must thank him for keeping some of my grander expectations in line with reality.

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### Abstract

— This report shows that gamma spectra identification by total flux correlation can be used to extend identification range over photo peak methods. Identification was based on two decision rules both employing cross-correlation coefficients. The largest coefficient (first decision rule) matched the unknown spectra with the correct source thirty-seven out of thirty-eight trials. The proposed likelihood function (second decision rule) had a success rate of thirty-five out of thirty-eight trials. These results were based on spectra generated by the transport code, Morse.

## I. INTRODUCTION

### Background

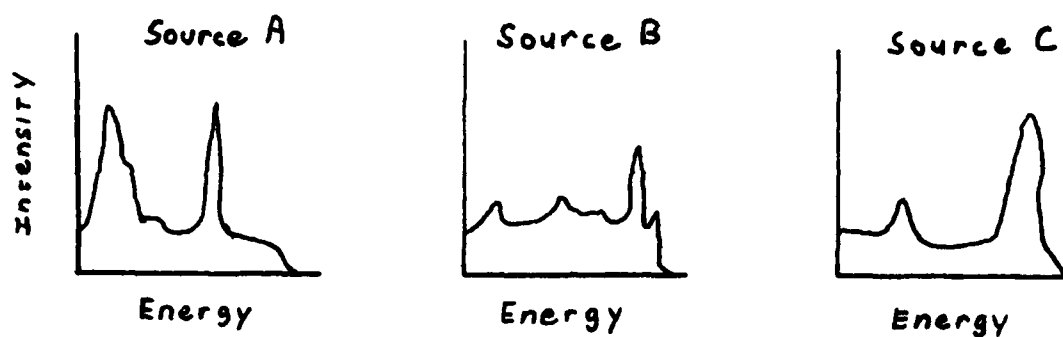
The analysis of characteristic gamma-ray spectra, from spontaneous nuclear decay, for identifying the atomic source or sources of radiation usually employs "matching" the unknown source's spectral features with the spectral features of known sources. Figure 1 illustrates this concept.

Figure 1, depicts the matching of the unknown source in a rather simplified manner. The match shown in part c shows the spectra being compared by their energy differences of the peaks and relative intensities of the peaks. The concept of the library and measured spectra will be expanded upon in the next chapter.

As illustrated in Figure 1, the spectral features that are most commonly used for matching are the peak lines in the spectra. The peaks are the easiest means for several reasons: (1) The energy of the peaks, or differences between peaks, can be determined; and (2) the relative intensities of the peaks can be determined.

The importance of knowing the relative intensity may not be entirely clear at this point. Basically, what if two sources containing different amounts of the same materials were measured? A proper match would then depend on the relative peak intensities, as well as line energies.

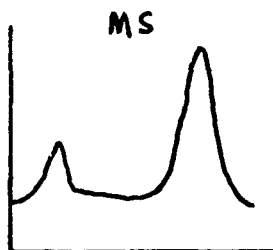
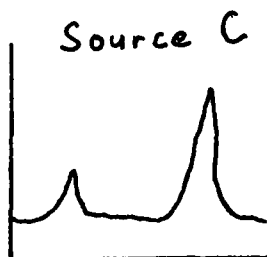
As a review, the interactions of photons with matter results in either partial or total loss of energy. The three major processes are (1) the photoelectric effect, (2) compton scattering by electrons in the



- (a) Spectra of known sources. These spectra will be referred to as the library spectra.



- (b) Spectrum of unknown source. This spectrum will be referred to as the measured spectrum.



- (c) The measured spectrum clearly matched source C of the library. Therefore, source C would be considered ms's source.

Figure 1. Illustrative example of "matching" a spectrum of some unknown source with a spectrum of a known source for identification.

atoms of the material, and (3) pair production. Compton scattering is the only process which does not absorb the entire energy of the photon. These processes occur in both the transport medium and detector material.

The peaks in a spectrum, like those in Figure 1, result from those gamma-rays which deposit all their energy only within the detector's active volume. The peaks produced by these uncollided gamma-rays are known as photopeaks. Typically these peaks dominate the spectral features and are used for source identification. The uncollided and collided photons are illustrated in Figure 2.

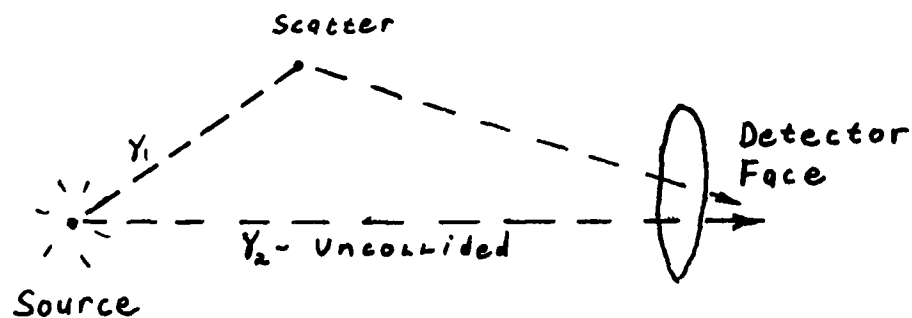


Figure 2. Collided and uncollided photons.

As the photons of a source are transported to further and further distances, the probability that a photon will interact in the transport medium increases, causing the ratio of collided to uncollided photons to continually increase. Eventually, there would be so few uncollided photons reaching the detector and depositing all their energy, that the identification of the peaks in the recorded spectrum would no longer be practical for identification purposes.



Now, if the photopeaks in the spectrum were the only information available in a gamma-ray spectrum, then isolation of these peaks become more and more difficult as detection range increases. Eventually the photopeaks would no longer be identifiable, since the intensity of the photons would decrease with distance. This suggests that the spectrum's information vanishes whenever the peaks vanish.

This degradation of peak lines can be seen in figures 3 and 4. Figures 3 and 4 show output spectra for source photons with energies of 1.0 Mev and .5 Mev, respectively. The transport of photon lines must be represented by the transport of energy groups and not of lines. The group structure is due to the use of multigroup cross-sections that use discrete energy groups. The 1.0 Mev photon is in the energy bin having limits of 1.0-.80 Mev while the .5 Mev source has bin limits of .45-.5 Mev. The energy bins, as given by the group widths of the cross-section library, are contained in Appendix A.

If the photopeaks were used as the only information in determining a match, identification would be limited by the ability to isolate and identify the photopeaks of the measured spectrum. This thesis takes the position that counting the uncollided as well as the collided photons might add "structure" to the spectrum unique for different sources. This added "structure" may then be used for extending the identification range.

### Objectives

The basic objective of this thesis is to model and study the remote detection of radioactive materials by measuring their naturally emitted gamma-rays, obtaining spectra, and then employ spectral analysis to

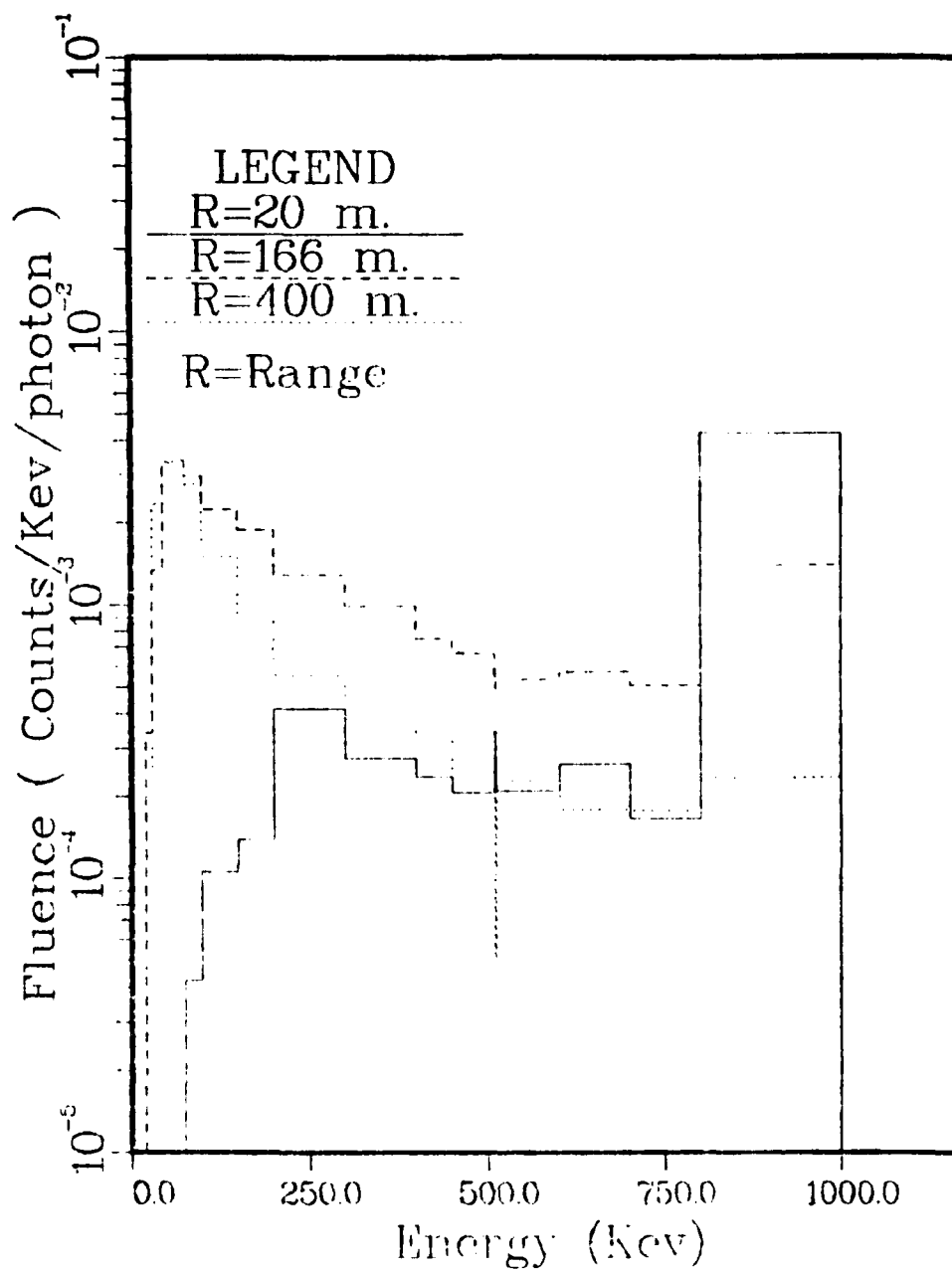


Figure 3. Fluence spectrum for a 1. MeV source in normal density air (1.298 g/l) at radii of 20 meters (.2 mfp), 166 meters (1.5 mfp), and 400 meters (3.0 mfp).

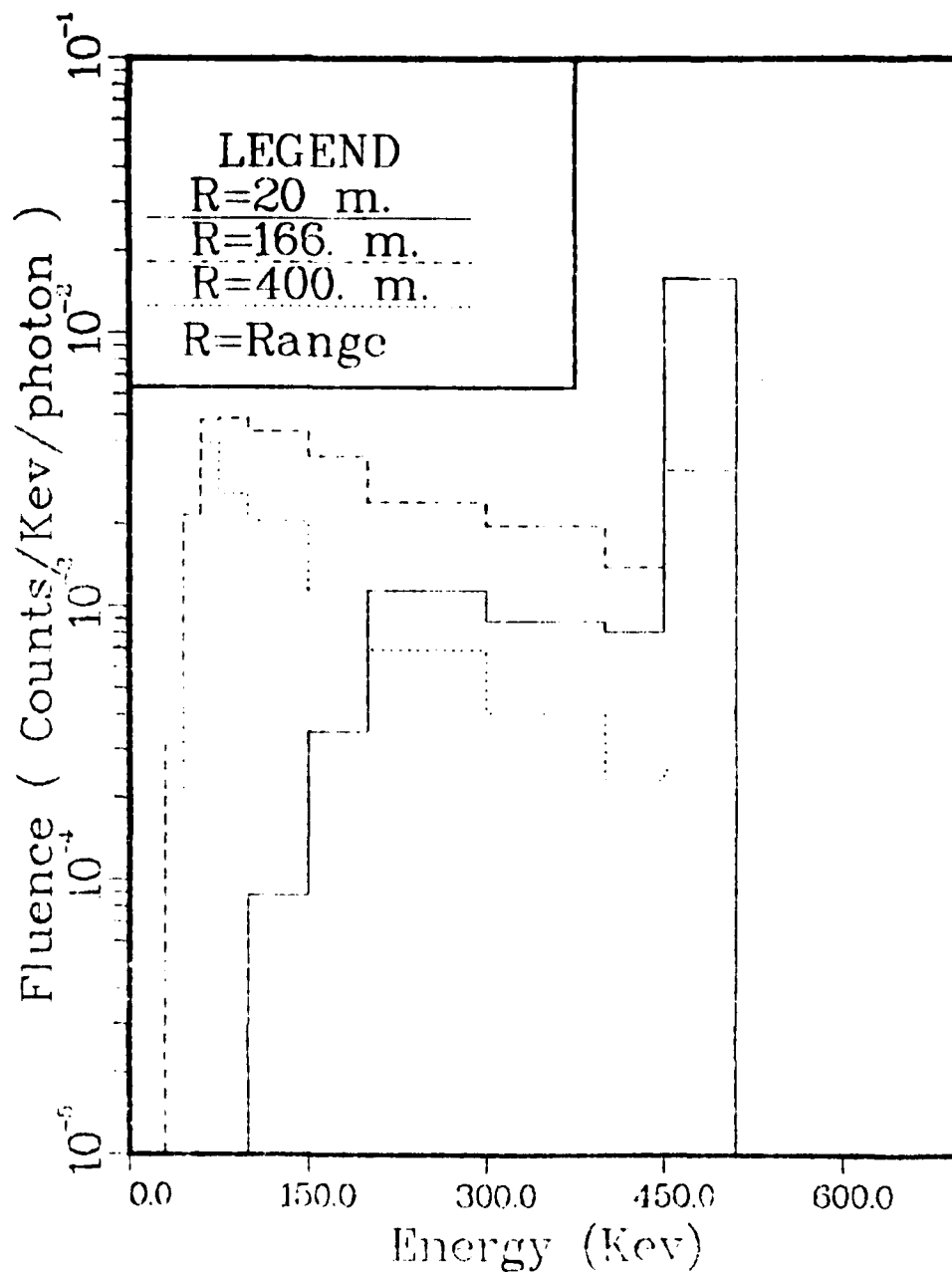


Figure 4. Fluence spectrum for a .5 MeV source in normal density air (1.298 g/l) at radii of 20 meters (.24 mfp), 166 meters (2.0 mfp), and 400 meters (3.9 mfp).

obtain the range limitations at which the spectra's sources can be determined.

This thesis studies the hypothesis that the total spectrum, consisting of both uncollided and collided photons, may be used as a means for source identification and, therefore, extend the identification range. In other words, do scattered photons contain usable information and if so, how can this information be extracted?

### Scope

The theoretical study lays the foundation for further work by presenting an identification technique, based upon proposed decision rules, which can be employed to select a source from its spectrum. This study will investigate and study the ability of the identification method to make the proper source selection as the photons are detected at increasing distances. In addition the question on whether a "new" source, not included in the library, has been detected will be investigated. The results will also be related to detection time.

To reach the objectives, two basic schemes were analyzed. The two scenarios were:

- (1) If the measured spectrum's source is known to be one of the library sources, how does increasing the detection range affect the ability of the decision rules to make the correct source identification? This scenario will consist of both a "good" and "poor" measurement for the measured spectrum. The meaning of "good" and "poor" will be explained in Chapter II.

- (2) If a measured source is not in the library, does the decision rules indicate that a non-library source is present? Again a "good" and "poor" measured spectrum will be used.

### Limitations of Problem

This thesis utilized a Monte Carlo transport code for the generation of all output spectra (Ref 1). Gamma-rays were transported in infinite (no ground or structural interaction), homogeneous air which consisted of 79%  $N_2$  and 21%  $O_2$ , by volume with a density of 1.298 g/l. The modeling of the photon transport, imposed the following conditions on the spectra outputs:

- (1) Sources were isotropic point sources.
- (2) A spherical detector, completely surrounding the point source was used to estimate fluence (See Figure 5). This assumption is necessary to reduce the number of transported photons to a reasonable number. The characteristics of the transported spectra should remain valid.
- (3) A time independent detection was modeled, with the number of source photons increasing or decreasing to model relative detection time. In other words, the fluence is estimated not the flux.
- (4) Background was not considered.
- (5) A realistic detector's response function was not modeled.

Each of these conditions imposed restrictions on the results and ultimately on the achievements of the objectives. A brief remark on each of these conditions follows.

Choosing an isotropic point source to model the configuration of the radioactive material was not a bad assumption. If source dimensions were no greater than 1 meter, even the shortest transport range, i.e. 20 meters, is much much greater than the source dimensions.

Condition 2 results from the use of a spherical detection surface surrounding the point source. Figure 5 illustrates this configuration.

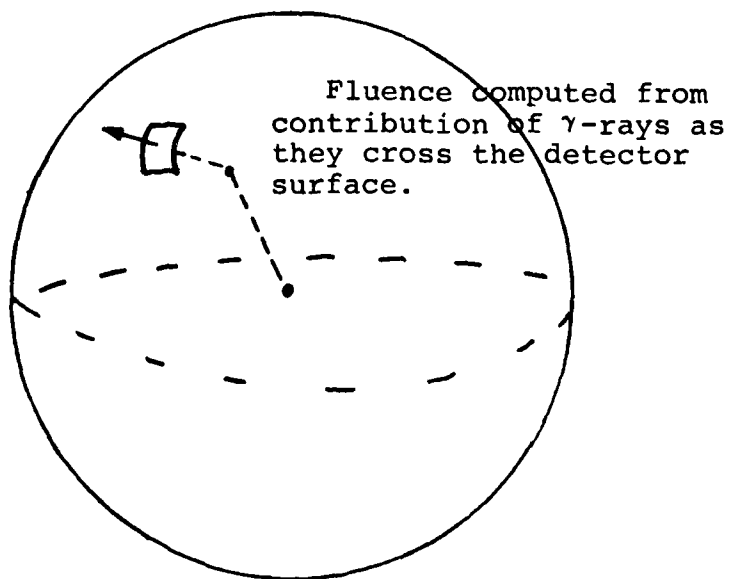


Figure 5. Geometrical arrangement used for estimating fluence of a point source with the Monte Carlo Code, Morse.

This setup was chosen due to it's ability to reduce the variance of the fluence estimations (Ref 2). This fluence estimation technique is one of several standard methods (Ref 2).

A time independent detection means that the code is not modeling continuous photon emissions from the source. Rather, the total intensity of the source spectra is determined by the number of photons that is input to the code for transport. The intensity of the bins in the source spectra is determined by specifying the relative amounts of the photons for each energy group.

The limitations imposed by conditions 4 and 5 contain the greatest deviations from a real detection.

Condition 4 is rather straight forward. No background was folded or artificially incorporated into the generated spectra. This lack of a background allows for a perfect environment to be modeled. Of course, this condition is never obtained in a real detection environment. This thesis assumes total background elimination is possible while at the same time not destroying the spectral structure created by the collided photon contributions. Remember, it is the scattered photon contributions to the spectra that is being investigated as a possible means of extending the identification range over the photopeak range.

Condition 5 also incorporates the ideal detector into the transport model. A response function of 1 for any photon crossing the detector surface was used. This ideal response indicates that the efficiency for all photons of any energy is the same. This condition is not true for a real detection case.

### Layout of Thesis

Chapter II introduces the sources used for the spectral analysis and discusses how they were constructed and transported.

Chapter III introduces the concept of spectral analysis by cross-correlation and a likelihood function. The concept of a library and measured spectra are defined and illustrated.

Chapter IV presents the results of the spectral analysis and illustrates a method for determining minimum detection time.

Chapter V follows with conclusions and recommendations.

The appendices of this thesis are located at the back of this paper and they are ordered in the same sequence in which they are mentioned.



## II. SOURCE TRANSPORT

### Source Considerations

The sources considered in this study were based on the following assumption:

Different compositions of radioactive nuclear materials emit gamma-radiation, that when detected close to their surface, give spectra of unique characteristics as do single isotopes.

Only three isotopes were considered: U-235, U-238, and Pu-239.

The source spectra were artificially constructed from combinations of these isotopes based upon several considerations:

- (1) The untransported source spectra should reflect, to some degree, the relative amounts of the combined isotopes.
- (2) The source spectra should have some major lines in common but with different relative intensities. This condition will help determine the ability of the analysis method to distinguish two different amounts of the same material.
- (3) Some scattering in the initial photon emissions from the isotope sources were factored into the spectra. This spread was so the source spectra would better model spectra at the surface of the material. It should be stated that a rigorous down-scatter analysis was not attempted because the design of the sources was deemed unnecessary for this theoretical study.

Even though the source spectra were constructed based upon these considerations, the details of the source spectra are somewhat arbitrary, since the analysis technique is being evaluated, not the detection of any specific sources.

The major energies of the isotopes are tabulated in Table I.

Table I. Major gamma-ray energies of radioactive material (Ref 3:4)

<u>Isotope</u>	<u>Energy (keV)</u>	<u>Intensity 1/(g-sec)</u>
U-235	185.72	4.3E4
U-238 *	1001.10	1.0E2
	766.4	3.9E1
Pu-239	129.28	1.4E5
	413.69	3.4E4

\* These energies actually arise from Pa-234m daughter of U-238.

Figures 6 through 8 indicate the source compositions used in this thesis. The figures are labeled source A, source B, and source C, respectively, and will be referred to as such throughout this thesis.

Table II gives the percentage basis of the source spectra.

While the energies for the sources were based upon Table I, the figures show group structure instead of individual line energies. Recall that the bin limits may be found in Appendix A.

All spectra were graphed, throughout the paper, with ordinate units of counts/keV/source photon vs. abscissa units of keV. The ordinate values are  $4\pi R^2$  times fluence for each energy group. Spectra plots were done in this manner, so that the spectra at different ranges could all be represented on the same scale.

#### Spectra Generation

All of the output spectra were artificially produced by the transport code. As such, estimates of the group fluence values (means) could be generated to the statistical degree desired by running the code as

# Source A

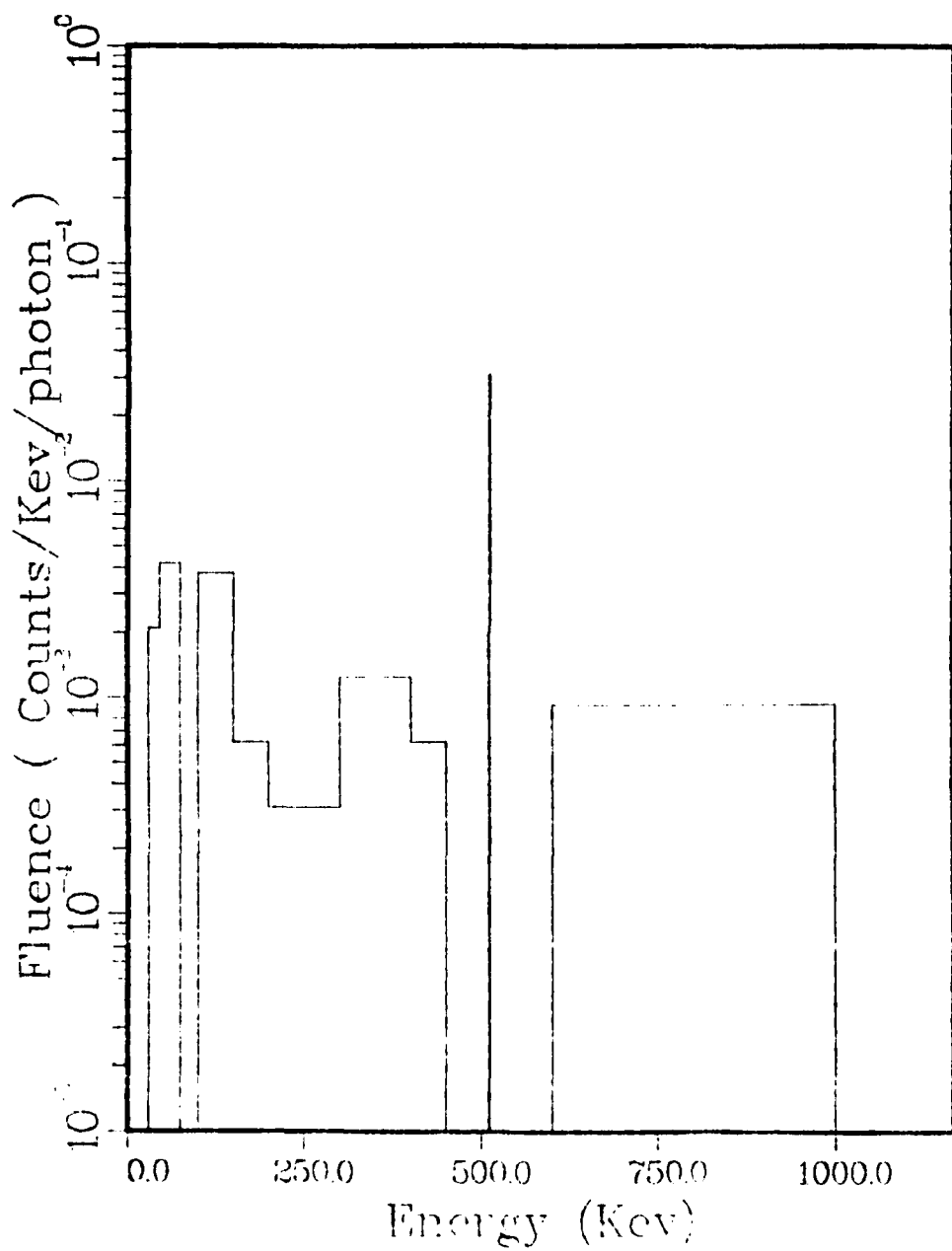


Figure 6. Fluence spectrum of Source A.

# Source B

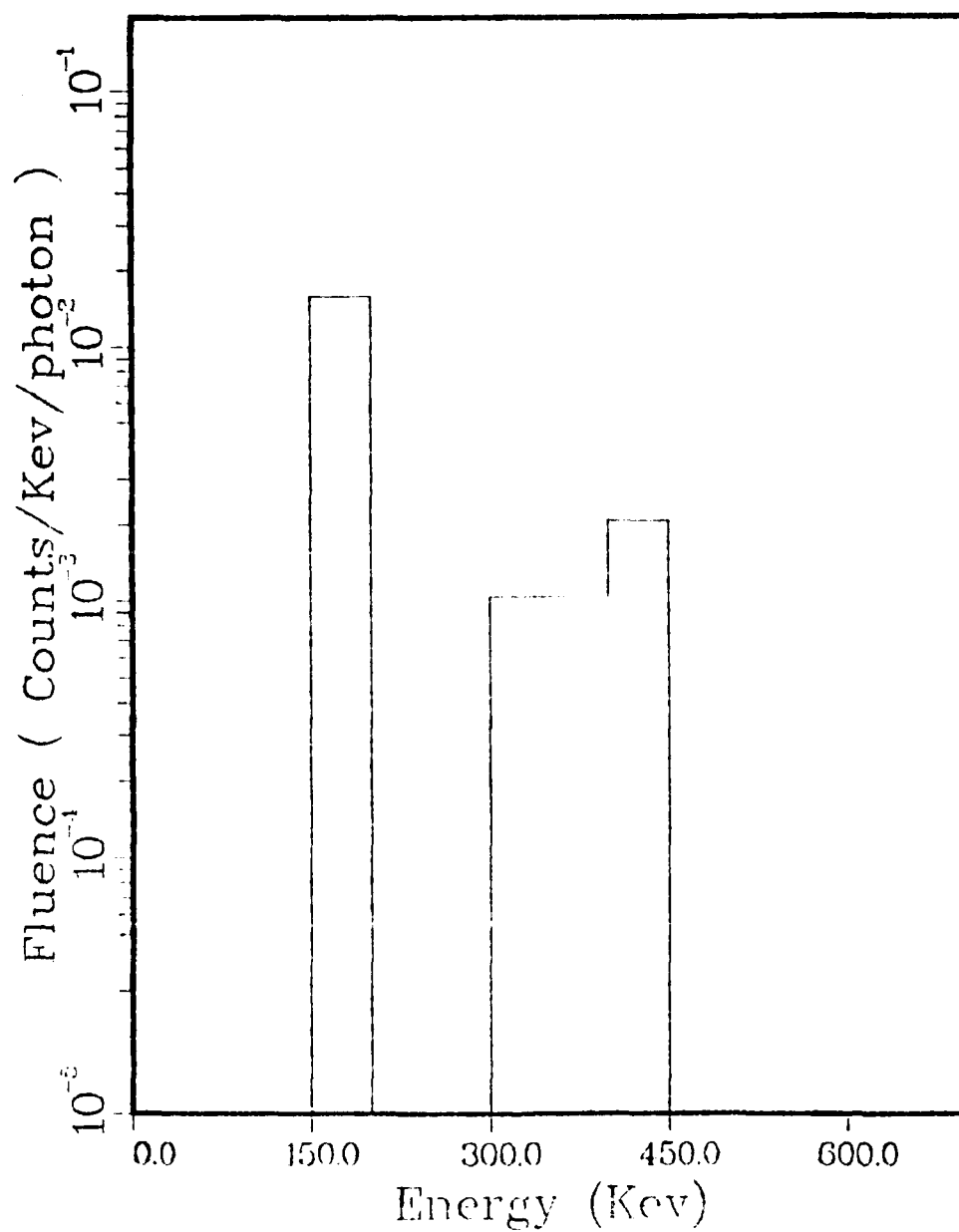


Figure 7. Fluence spectrum of Source B.

# Source C

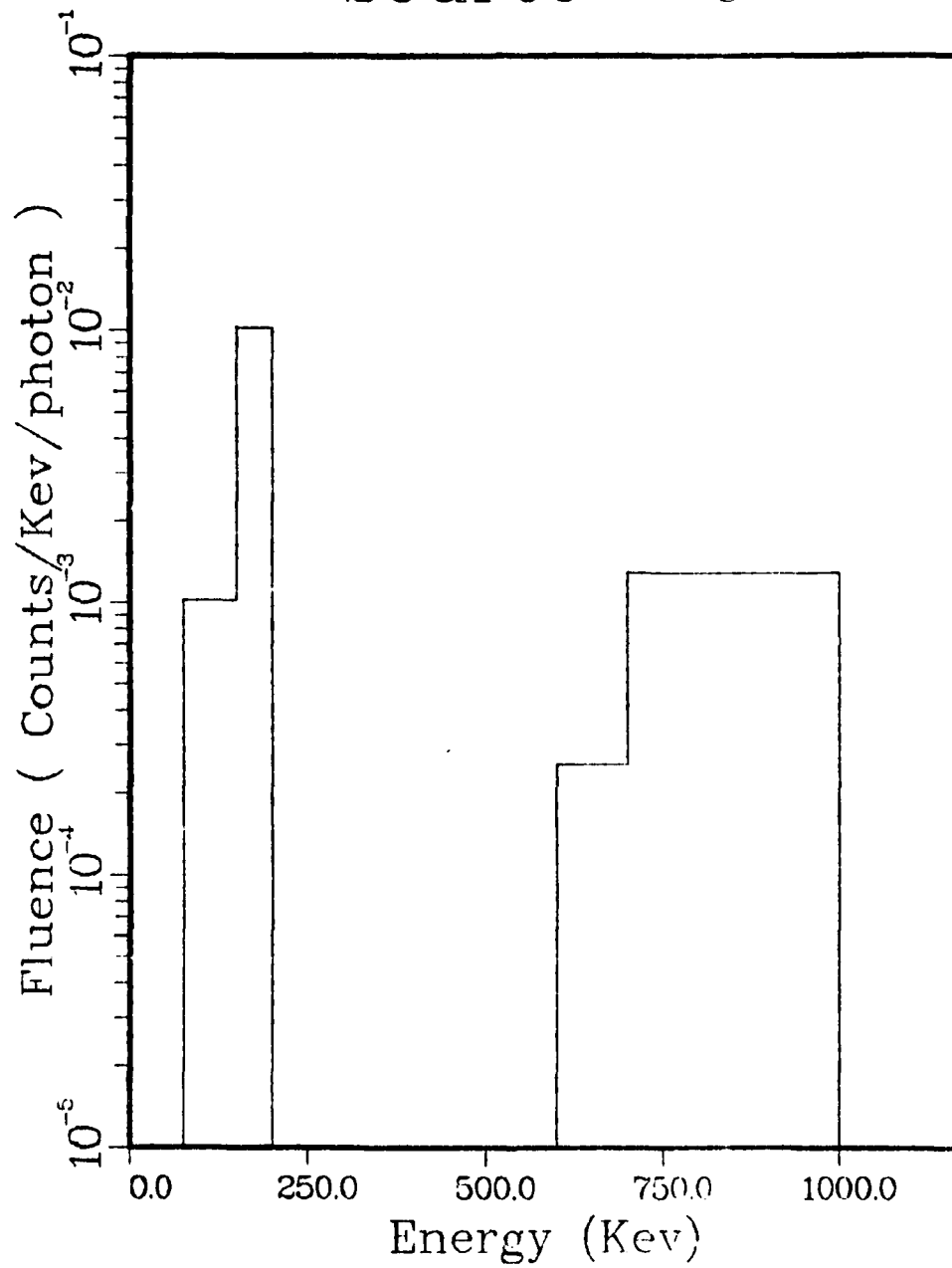


Figure 8. Fluence spectrum of Source C.

Table II. Spectra intensities of Sources on a percentage basis.

<u>Group (KeV)</u>	<u>Percent Intensity</u>		
	<u>Source A</u>	<u>Source B</u>	<u>Source C</u>
1300	.0	.18E-4	.0
1000	.06974	.0	.2564
800	.0349	.717E-5	.1282
700	.3489	.0	.0256
600	.0	.0	.0
512	.2324	.0	.0
500	.0	.0	.0
450	.0177	.1043	.0
400	.0465	.1043	.0
300	.0116	.0	.0
200	.1164	.7914	.5129
150	.0698	.0	.0513
100	.0	.0	.0256
75	.0232	.0	.0
60	.0233	.0	.0
45	.0116	.0	.0
30	.0	.0	.0
20	.0	.0	.0
10	.0	.0	.0

many times as needed. Recall that the more samples that are used, the better the true mean estimate can be found, i.e. the variance of the mean is proportional to  $1/N$ , where  $N$  is the number of estimated values for the quantity of interest (for example fluence). With this in mind the output spectra were grouped into two categories.

The two categories were the library spectra and the measured spectra. These categories have been previously mentioned and will now be fully defined.

The library spectra were generated from the sources in a manner that allowed for obtaining an estimation of the true fluence means. That is to say, the library spectra were generated by averaging the fluences of many runs. This resulted in a standard deviation of the mean being obtained with each group fluence (Remember the energy bin structure). Of course, if the sources were available for a real detection, then careful measurement of these sources under controlled conditions would be carried out. This is what the library spectra represents.

In contrast, the measured spectra were generated so as to represent a measurement under less than optimum conditions. Hence, their fluences would be expected to be randomly distributed about the mean fluences of the library spectra; an exact match was not likely.

The reasons for generating spectra at different ranges will be explained shortly.

The measured spectra were further broken up into two types: the "good" and "poor" measured spectra.

The terms "good" and "poor" were used only to indicate that the "good" spectra were generated with a larger number of source photons

than the "poor" spectra, thus the "good" spectra should match the library spectra more closely. The term "poor" does not indicate that the generation of the "poor" spectra resulted from computational error.

The method for generating these measured spectra are given below under the appropriate subheading.

Library Spectra. The library spectra consist of the three sources transported to 8 different distances: 20, 41, 83, 124.5, 166, 245, 333, and 390 meters. Each output spectrum was determined by making 3 runs with ten batches/run of 1000 source photons/batch. The group fluence values for the 3 runs were averaged and the fractional standard deviations were determined by batch statistics as given by the following:

Fluence Average (Multiple runs)

$$\bar{F} = \frac{\sum_{i=1}^N \bar{F}_i C_i}{\sum_{i=1}^N C_i}$$

N = number of runs

C<sub>i</sub> = number of batches for run i

$\bar{F}_i$  = average value of interest for run i

$\bar{F}$  = averaged value of interest (fluence)

Fractional Standard Deviation (Multiple runs)

$$f.s.d. = \frac{\sqrt{\sigma_{\bar{F}}^2}}{\bar{F}} = \sqrt{\frac{\sum_{i=1}^N C_i^2 \sigma_{\bar{F}_i}^2}{\sum_{j=1}^N C_j^2}} \frac{1}{\bar{F}}$$

Standard Deviation of the Mean (Any single run)



$$\sigma_{\bar{F}}^2 = \frac{1}{(N-1)} \left[ \frac{1}{n} \sum_{i=1}^N n_i F_i^2 - \frac{1}{n^2} \left[ \sum_{j=1}^N n_j F_j \right]^2 \right]$$

where,

$N$  = number of batches (note the change from multiple runs),

$n$  = total number of independent histories (function of the number of source photons),

$n_i$  = number of independent histories in  $i$ -th batch,

$F_i$  = accumulated estimate in  $i$ -th batch.

Note that:

$$n = \sum_{i=1}^N n_i$$

$$F_i = \frac{1}{n_i} \sum_{j=1}^N F_{i,j}$$

where  $F_{i,j}$  is the estimate from the  $j$ -th history in the  $i$ -th batch,

$$\bar{F} = \frac{1}{n} \sum_{j=1}^N n_j * F_j \quad (\text{for one run})$$

where  $\bar{F}$  is the mean, averaged over  $n$  histories.

The fractional standard deviation is

$$\text{f.s.d.} = \sqrt{\sigma_{\bar{F}}^2 / \bar{F}}$$

These statistics are computed by an analysis subroutine within the transport code (Ref 4).

So, if the desired f.s.d. is not low enough (f.s.d. might be higher for some bins than others) another run may be done in order to reduce the f.s.d. and therefore, reduce  $\sigma_{\bar{F}}$  giving a better estimate for the fluence means.

Why were the library spectra generated for different distances? These range generations were done so that the concept of using total spectra analysis, i.e. uncollided and collided spectral features, could be carried out. Figure 9 best illustrates this concept.

By "transporting" the library spectra at the same range as a measured spectra a match can be attempted using the total spectrum with the hope for extending the identification range.

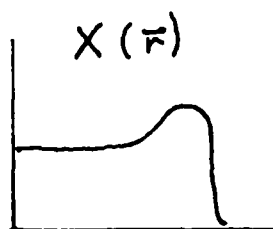
The purpose of the library spectra was to act as the spectra, corresponding to known sources, by which a match with a measured spectrum would be attempted. A perfect match would indicate that the measured spectrum was from the same source that generated the library spectrum. But this perfect match is not likely and only a best match would be expected.

This type of procedure allows for the investigation of the hypothesis that the scattered photons contain useful information.

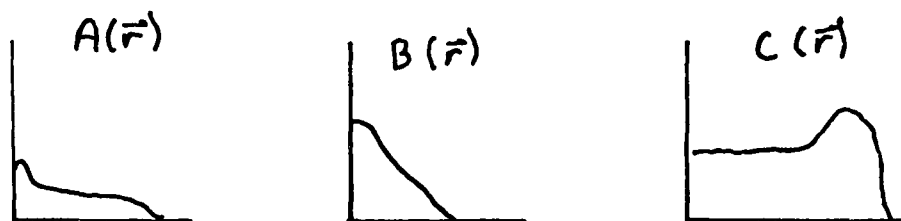
Measured Spectra. The measured spectra consist of the sources transported to the same distances as the library, but with fewer source photons. The purpose of the measured spectra was to act as the unknown sources for a match with the library spectra. The measured spectra's sources will be referred to as unknown sources. Of course, the unknown sources are known to us, so that conclusions on spectral analysis can be evaluated.



- (a) Source spectra (A-C) and unknown spectra X at the surface of their material. Can see that spectrum X "matches" spectrum C better than the other source spectra.



- (b) For some reason source X can not be measured at a close distance but only at range  $r$ . It is now difficult to tell which source X is.



- (c) By generating the library spectra at the same range a match may become possible. Now, spectrum C, at  $r$ , seems to match spectrum X at  $r$ .

Figure 9. Illustrative example for generating transported library spectra.

Also included was Source X, which was a source not used in the library. Source X was used so scenario (2) (Chapter I) could be answered. Source X is shown in Figure 10.

Two sets of measured spectra were generated; one referred to as the "good" spectra and the other referred to as the "poor" spectra. Only one run with one batch was used in their generation. Therefore, no estimate of the mean for each group fluence value was possible. What these measured spectra gave were fluence values randomly distributed about the means of the library values.

"Good" Spectra. These measured spectra were generated by using 1000 photons as the makeup of the source's intensity.

"Poor" Spectra. These measured spectra were generated by using 100 photons as the makeup of the source's intensity.

It should not be surprising to expect that the "good" spectra will be distributed with less variance about the means of the library (of the same source) than would the "poor" spectra. Table III clearly shows that this statement is correct. Table III gives the number of standard deviations away the measured fluences of source A are from the library fluences of source A at the range of 20 meters. Clearly, Table III (For the complete listing see Appendix C) shows that the standard deviations of the "poor" spectra are larger than the "good" spectra, on the average. As range increases the differences between the "good" and "poor" spectra become even more apparent.

The purpose for generating two different measured spectra for each source was to find out what the analysis procedure reveals when the search spectrum consist of either a good measurement (good statistics)

# Source X

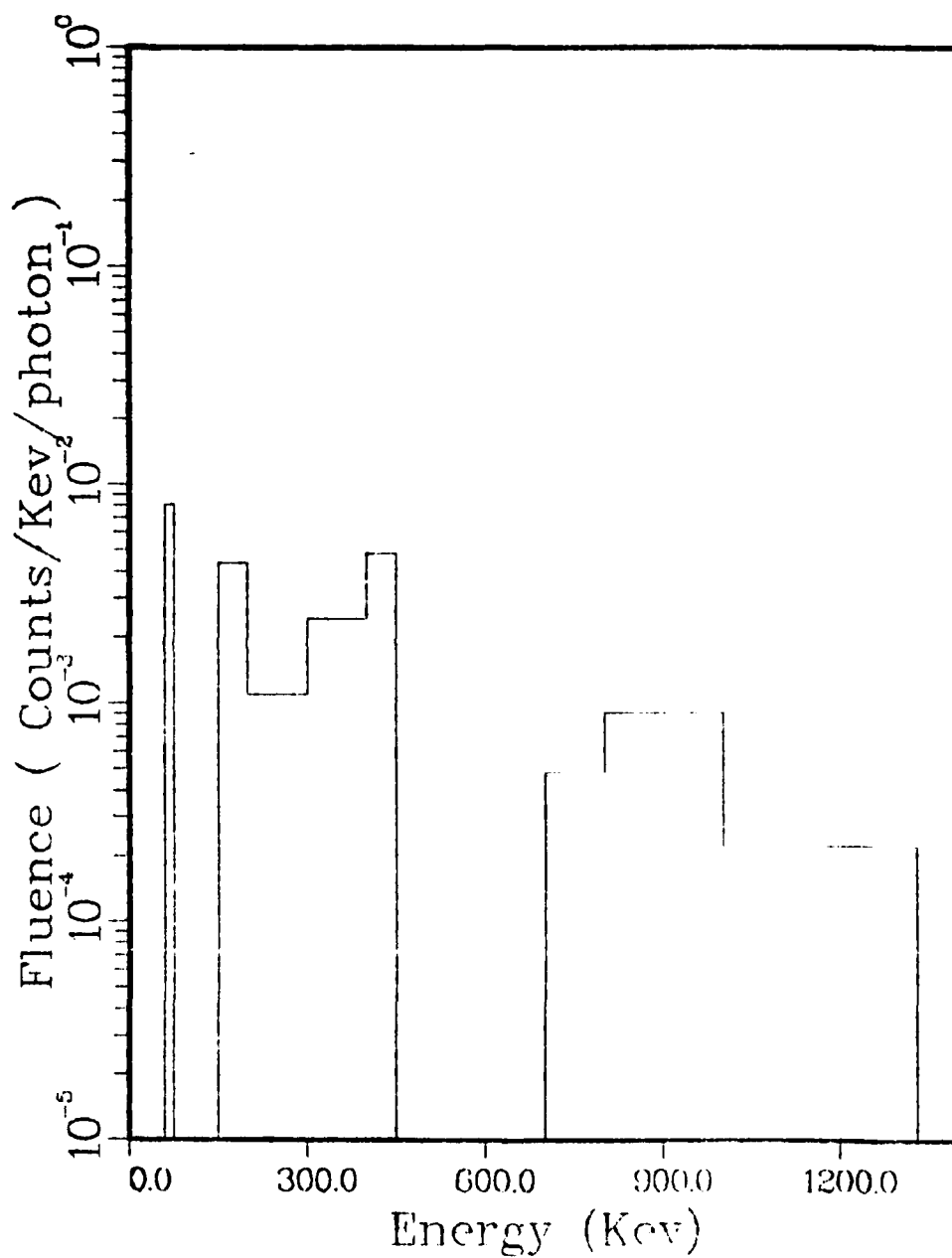


Figure 10. Fluence spectrum of Source X.

Table III. Distribution of "Good" and "Poor" fluences about the means of their library fluences. Source is Source A at the range of 20 meters. Values are in Counts/Photon.

<u>Group (KeV)</u>	<u>Library A (s. dev)*</u>	<u>MA (Good) (d.f.m.)**</u>	<u>MA(Poor) (d.f.m.)**</u>
1300	.0 (.0)	.0 n/a	.0 n/a
1000	5.94E-2 (7.47E-3)	6.195E-2 (-.34)	4.66E-2 (1.713)
800	3.043E-2 (6.24E-3)	3.548E-2 (-.81)	1.541E-2 (2.406)
700	2.947E-1 (1.77E-2)	2.894E-1 (.301)	2.528E-1 (2.369)
600	1.161E-2 (5.98-3)	9.587E-3 (.339)	0.0 n/a
512	1.811E-1 (8.10E-2)	1.808E-1 (3.3E-3)	1.539E-1 (.335)
510	1.866E-2 (6.23E-3)	1.551E-2 (.5059)	0.0 n/a
450	2.472E-2 (7.87E-3)	1.635E-2 (1.064)	5.716E-3 (2.415)
400	7.068E-2 (1.37E-2)	6.534E-2 (.0389)	1.087E-1 (-2.77)
300	6.154E-2 (1.512E-2)	5.293E-2 (.5696)	7.729E-3 (3.56)
200	1.183E-1 (1.764E-2)	1.404E-1 (-1.25)	1.491E-1 (-1.74)
150	9.528E-2 (1.63E-2)	7.627E-2 (1.162)	1.094E-1 (-.859)
100	1.492E-2 (8.72E-3)	3.563E-2 (-2.37)	2.235E-2 (.8510)
75	2.083E-2 (5.46E-3)	1.439E-2 (1.180)	1.309E-2 (1.418)
60	2.531E-2 (7.33E-3)	1.687E-2 (1.151)	3.929E-2 (-1.90)
45	1.524E-2 (6.37E-3)	6.451E-3 (1.378)	3.507E-3 (1.839)
30	3.897E-4 (9.048E-4)	0.0 n/a	0.0 n/a
20	0.0 n/a	0.0 n/a	0.0 n/a
10	0.0 n/a	0.0 n/a	0.0 n/a

\* (standard deviation of library (counts/photon))

\*\* (deviations from mean of library)

or poor measurement (bad statistics).

The library spectra are shown in Figures 11 through 13 at various distances.

The "good" and "poor" spectra can be found in Figures 14 through 21.

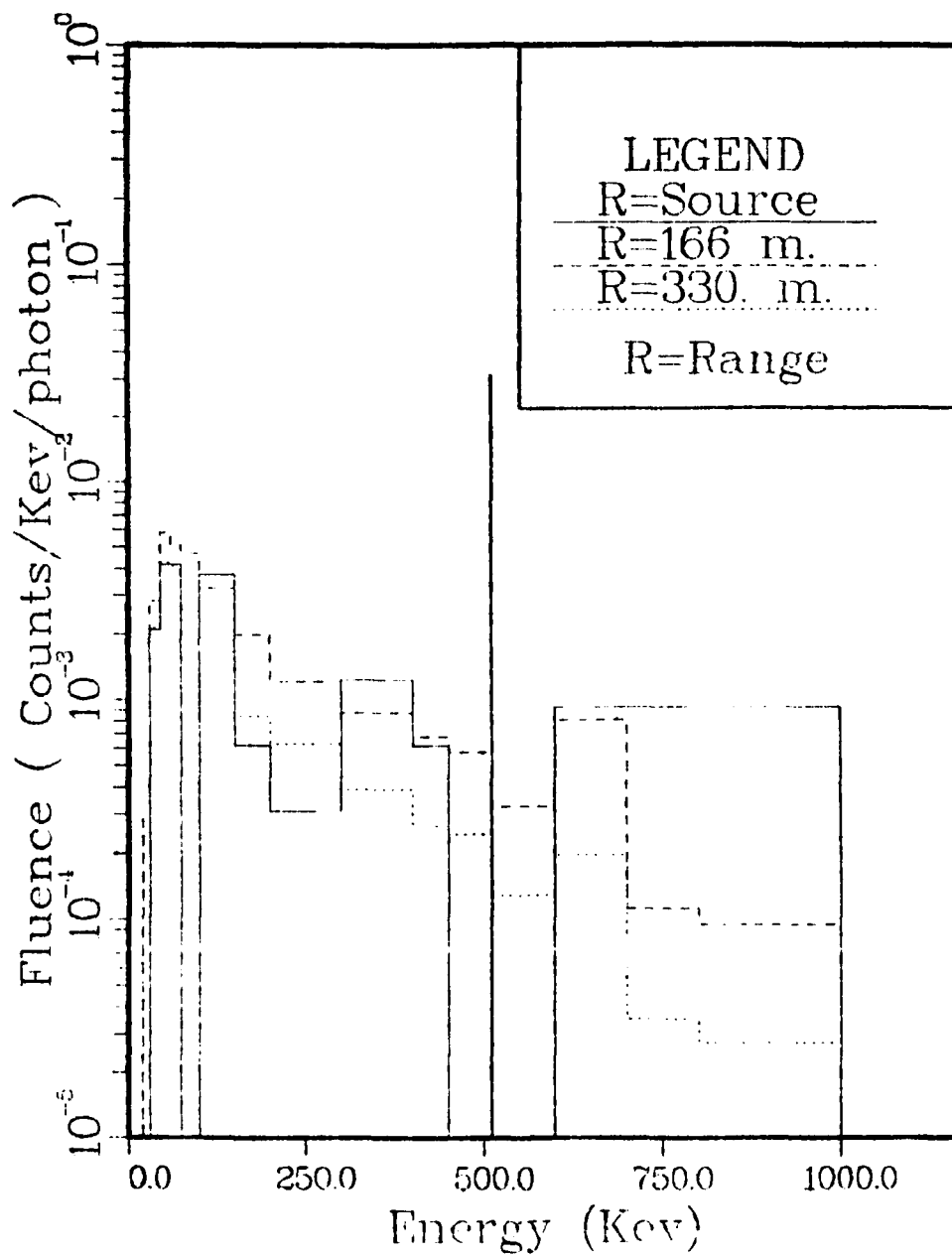


Figure 11. Library spectra of Source A at ranges of 166 meters and 330 meters.



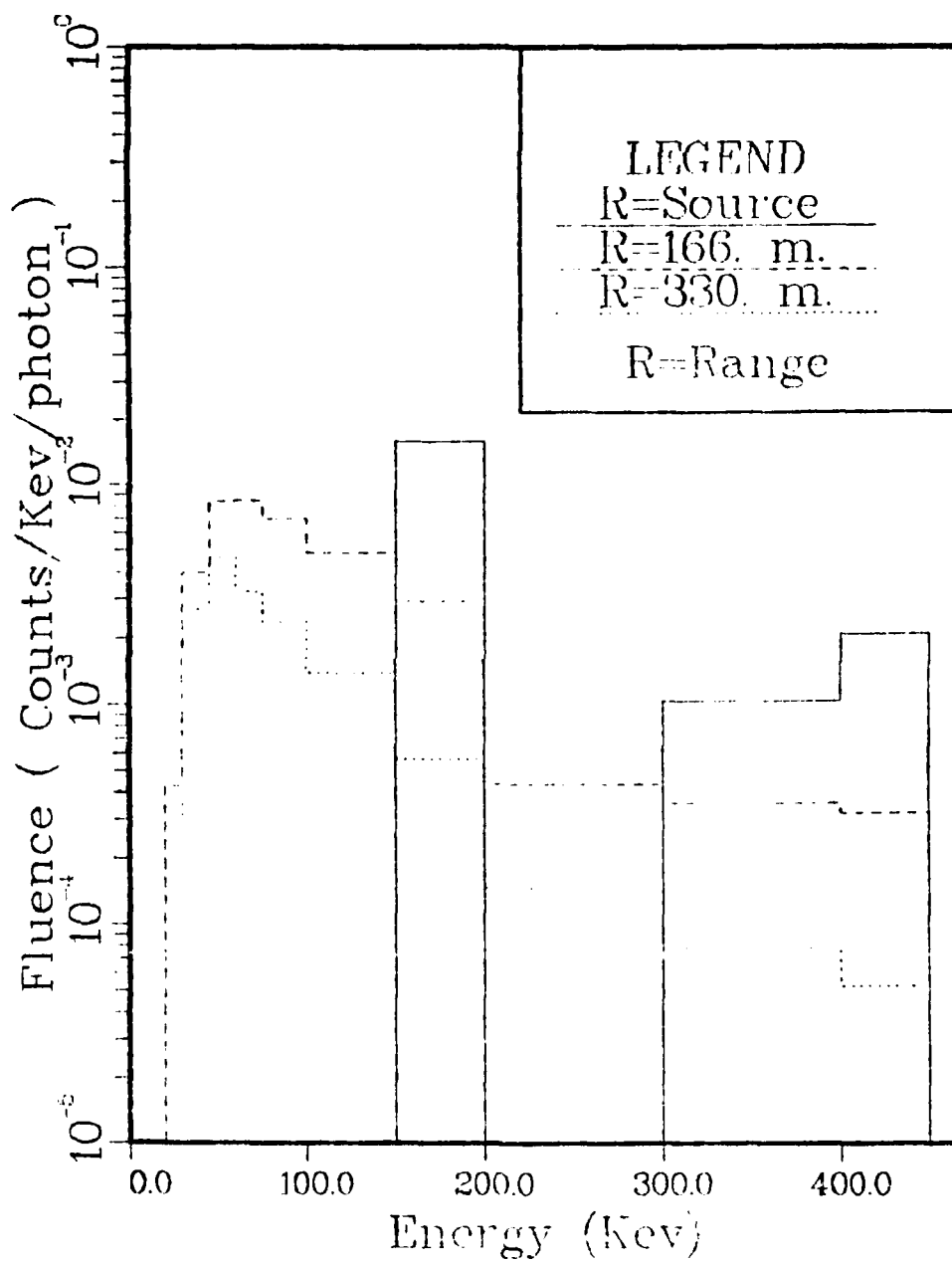


Figure 12. Library spectra of Source B at ranges of 166 meters and 330 meters.

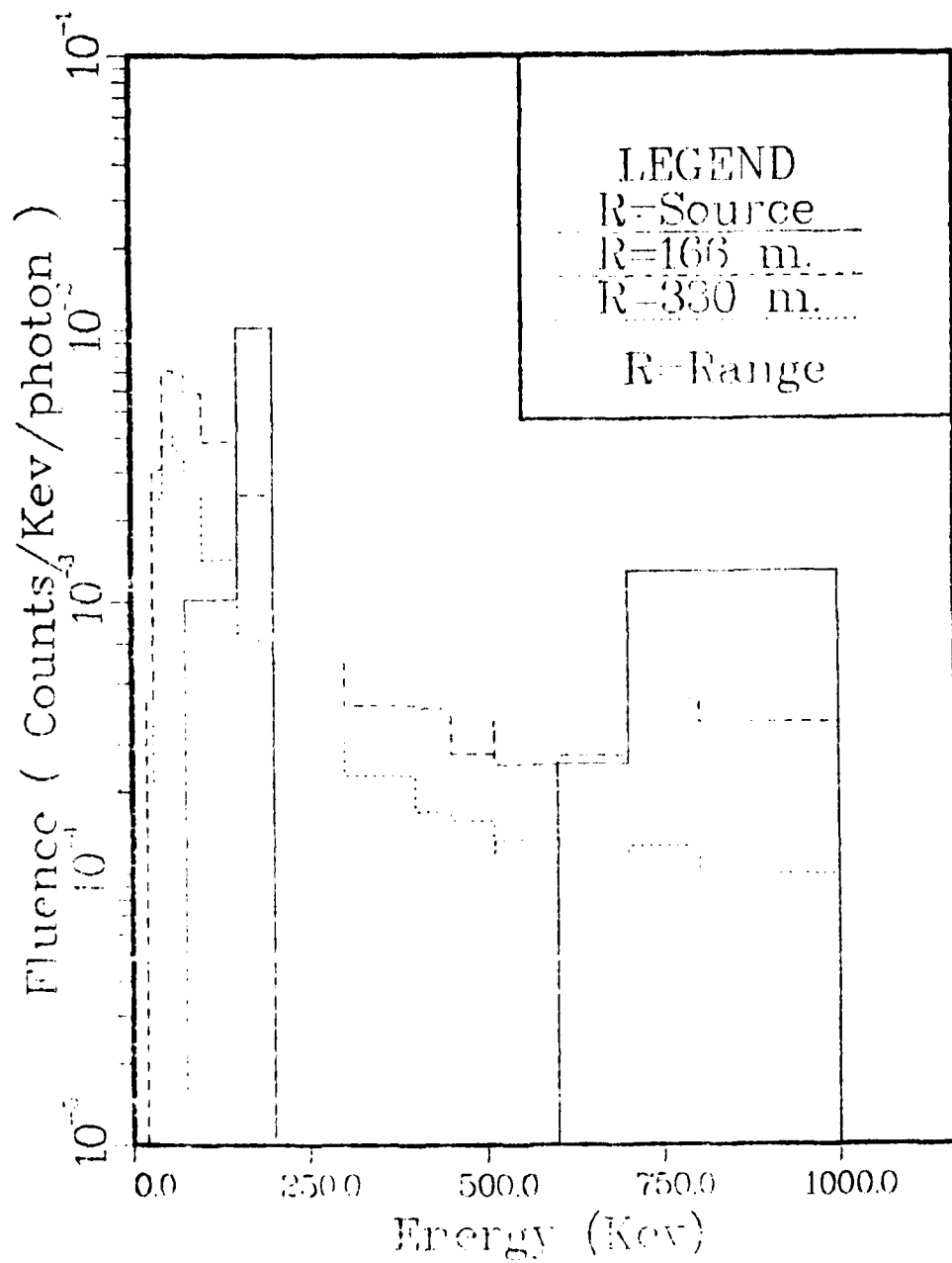


Figure 13. Library spectra of Source C at ranges of 166 meters and 330 meters.

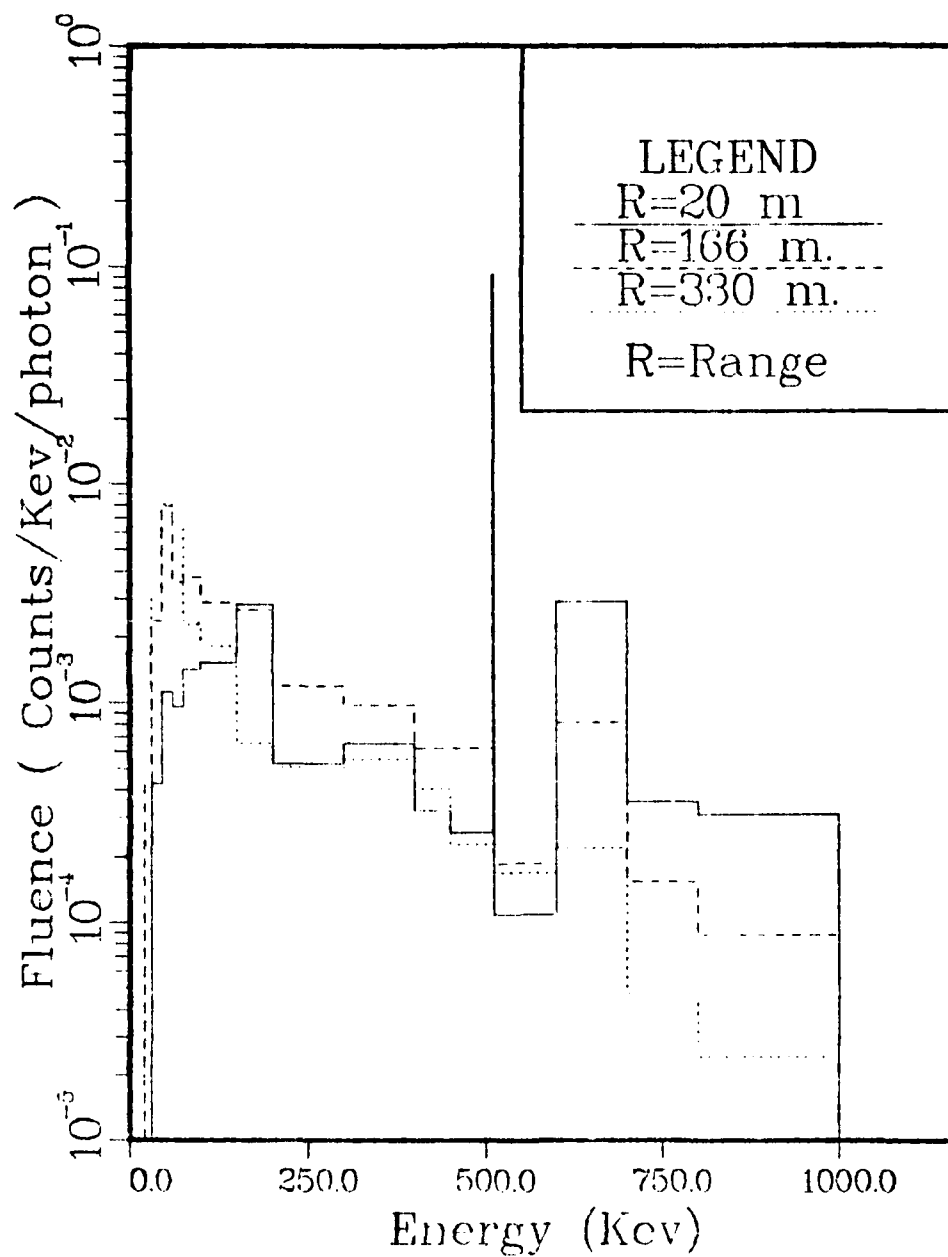


Figure 14. "Good" spectra of Source A at ranges of 20, 166, and 330 meters.

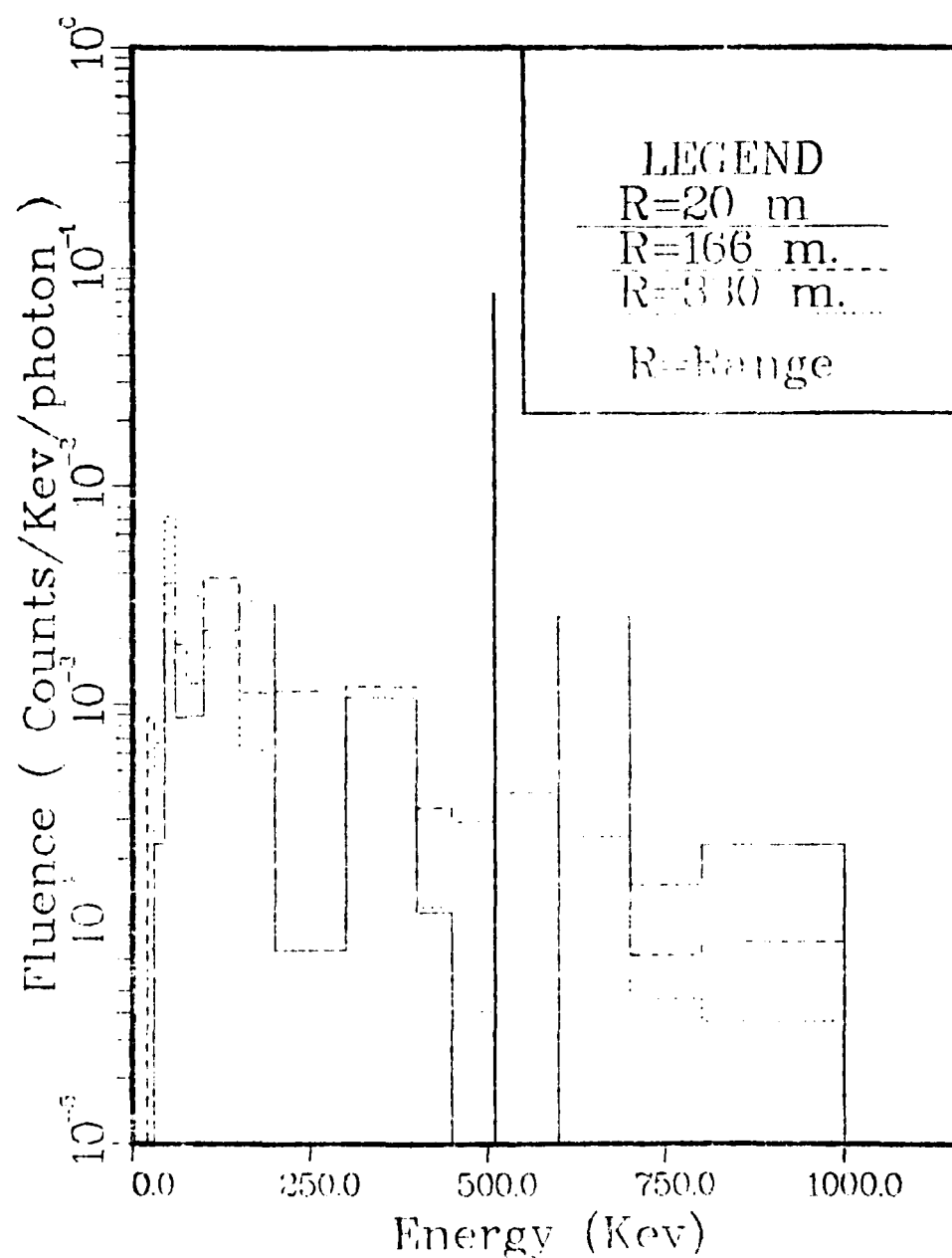


Figure 15. "Poor" spectra of Source A at ranges of 20, 166, and 330 meters.

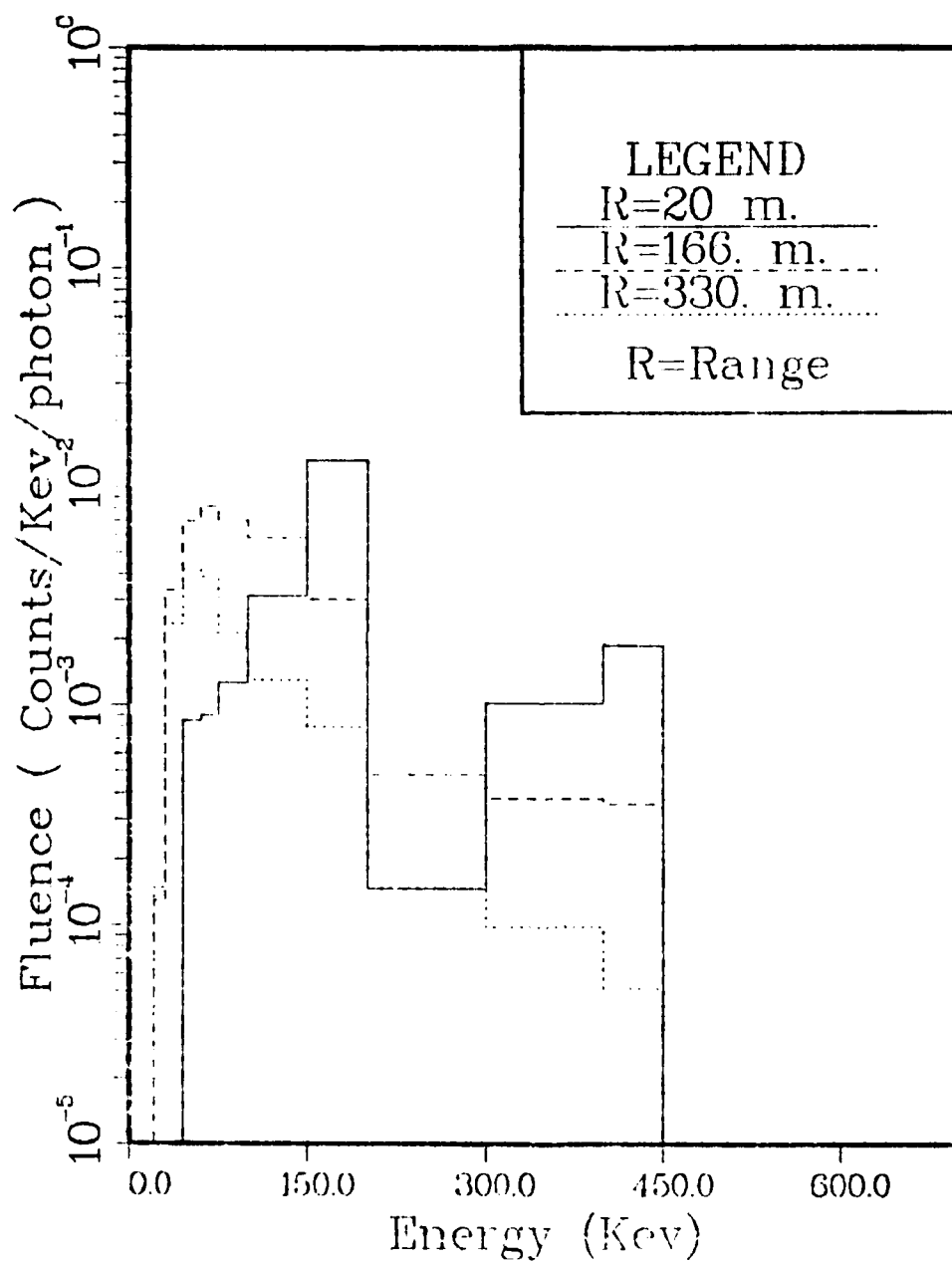


Figure 16. "Good" spectra of Source B at ranges of 20, 166, and 330 meters.

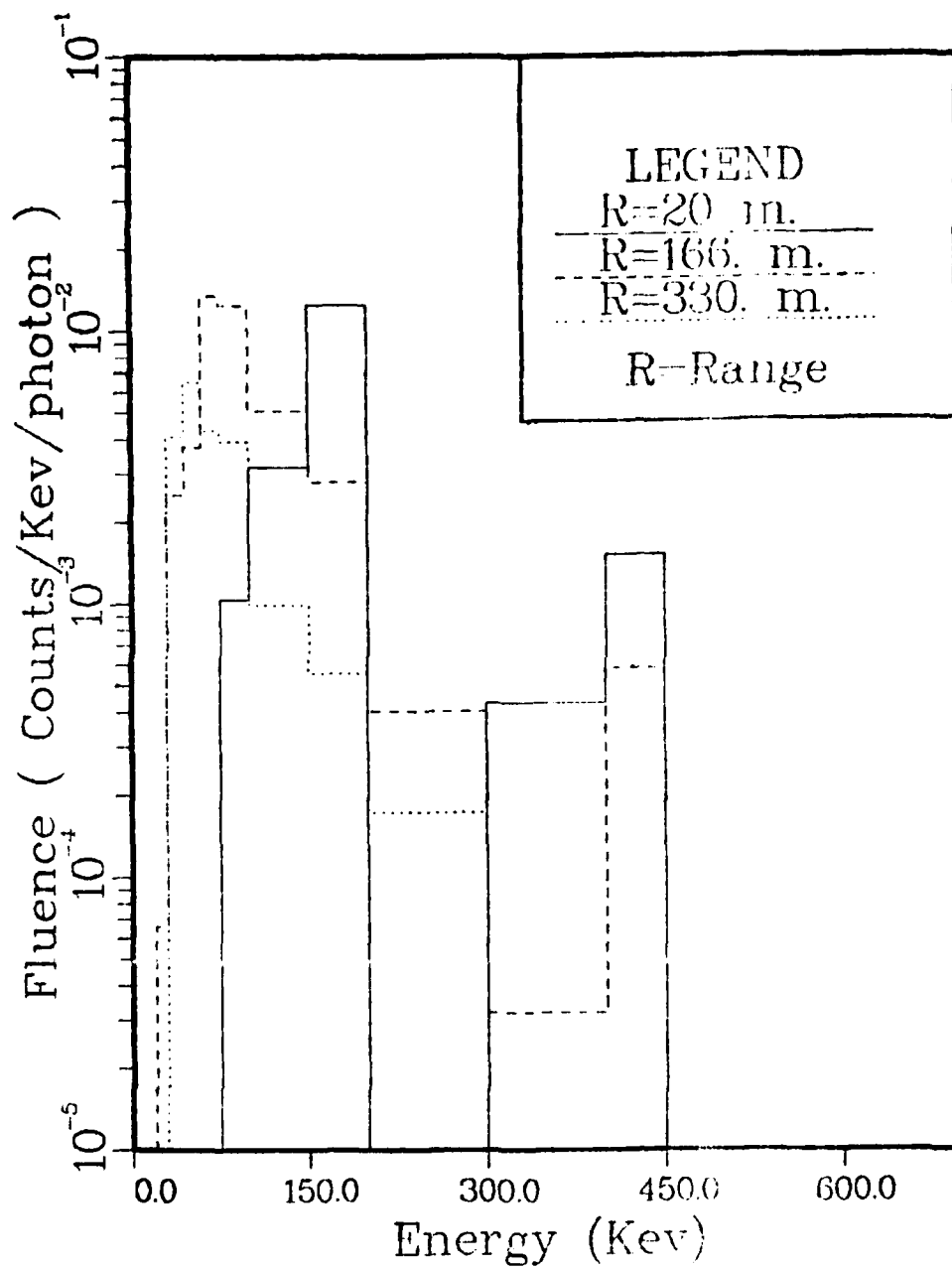


Figure 17. "Poor" spectra of Source B at ranges of 20, 166, and 330 meters.

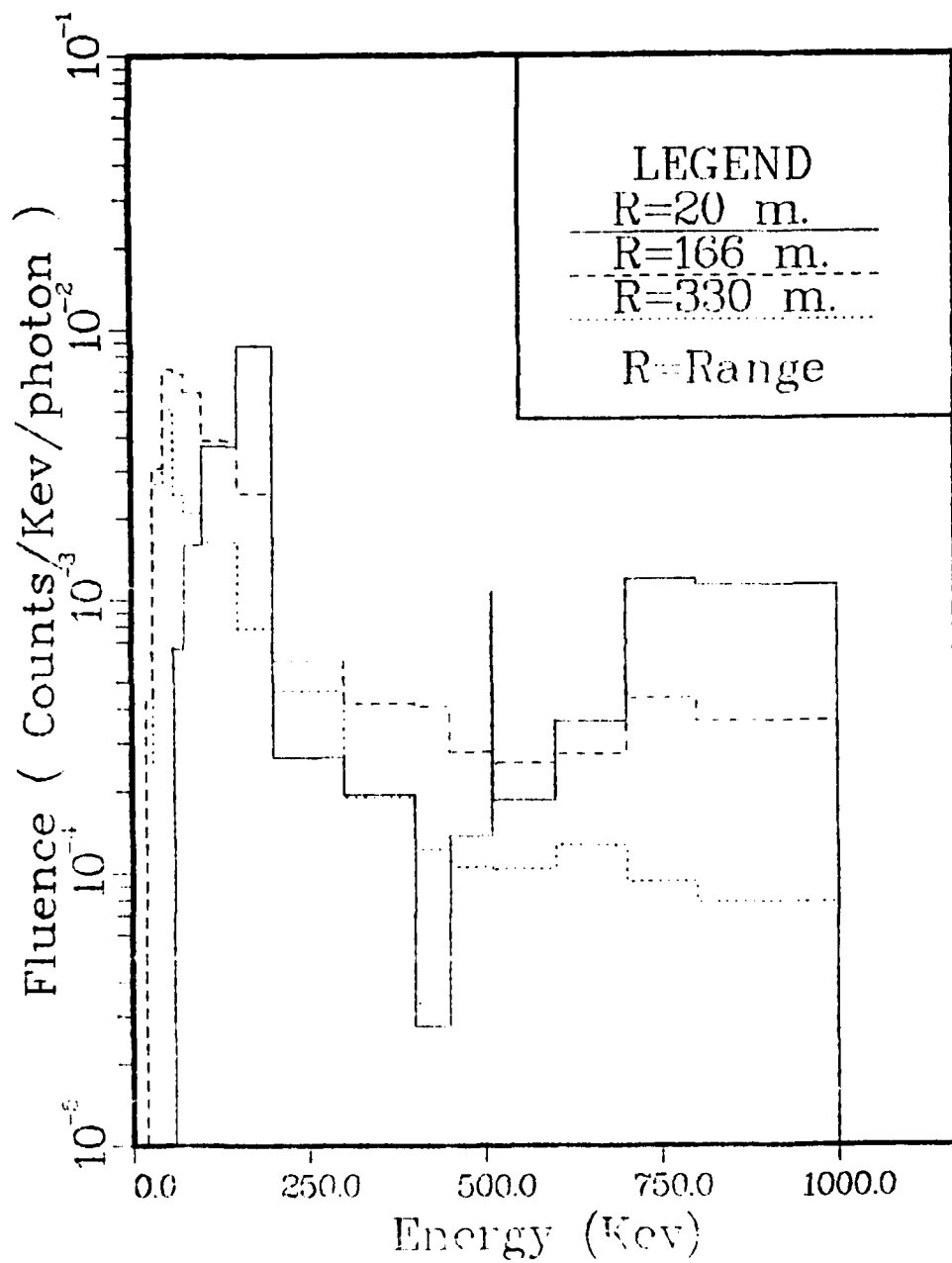


Figure 18. "Good" spectra of Source C at ranges of 20, 166, and 330 meters.

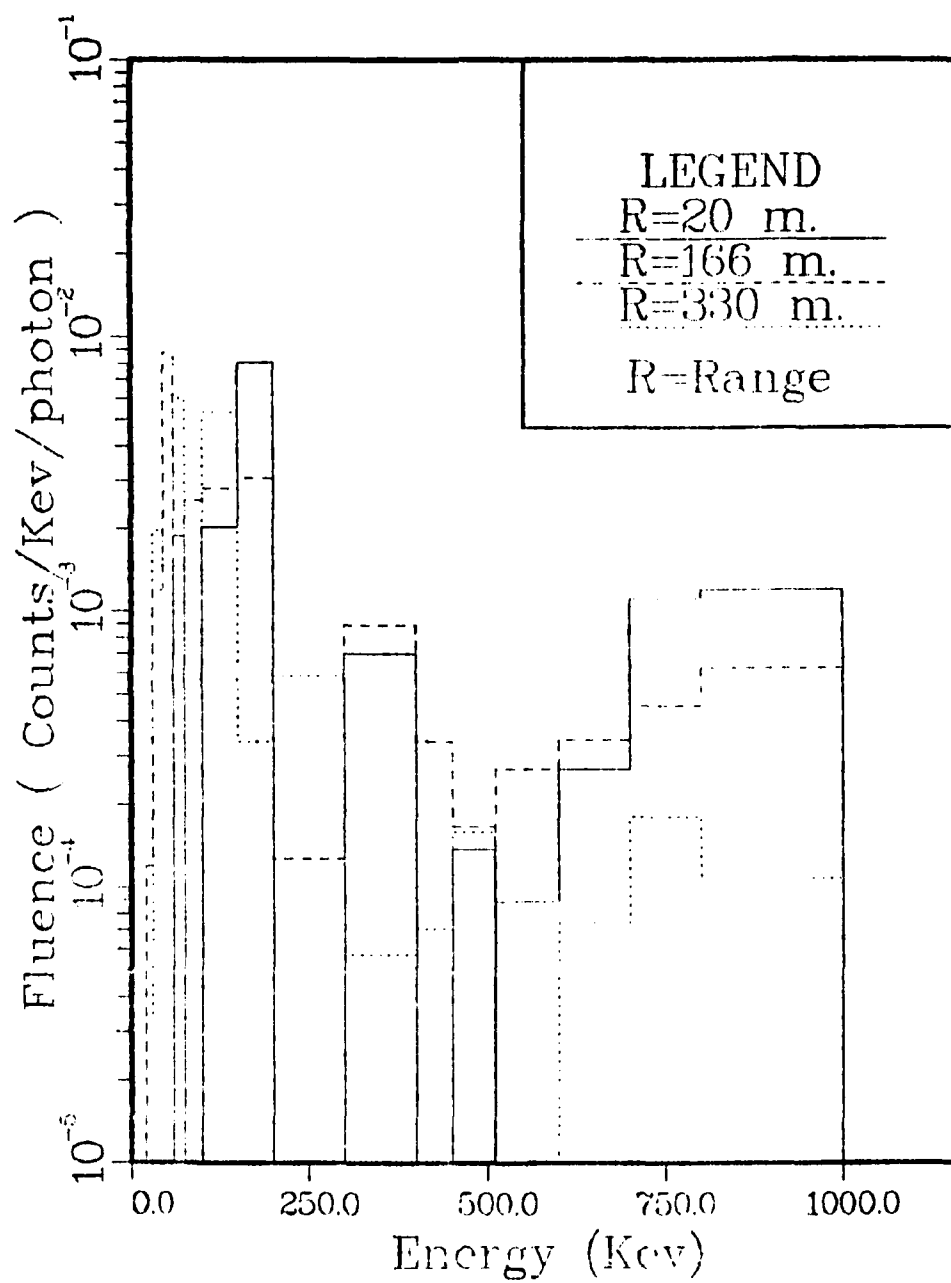


Figure 19. "Poor" spectra of Source C at ranges of 20, 166, and 330 meters.



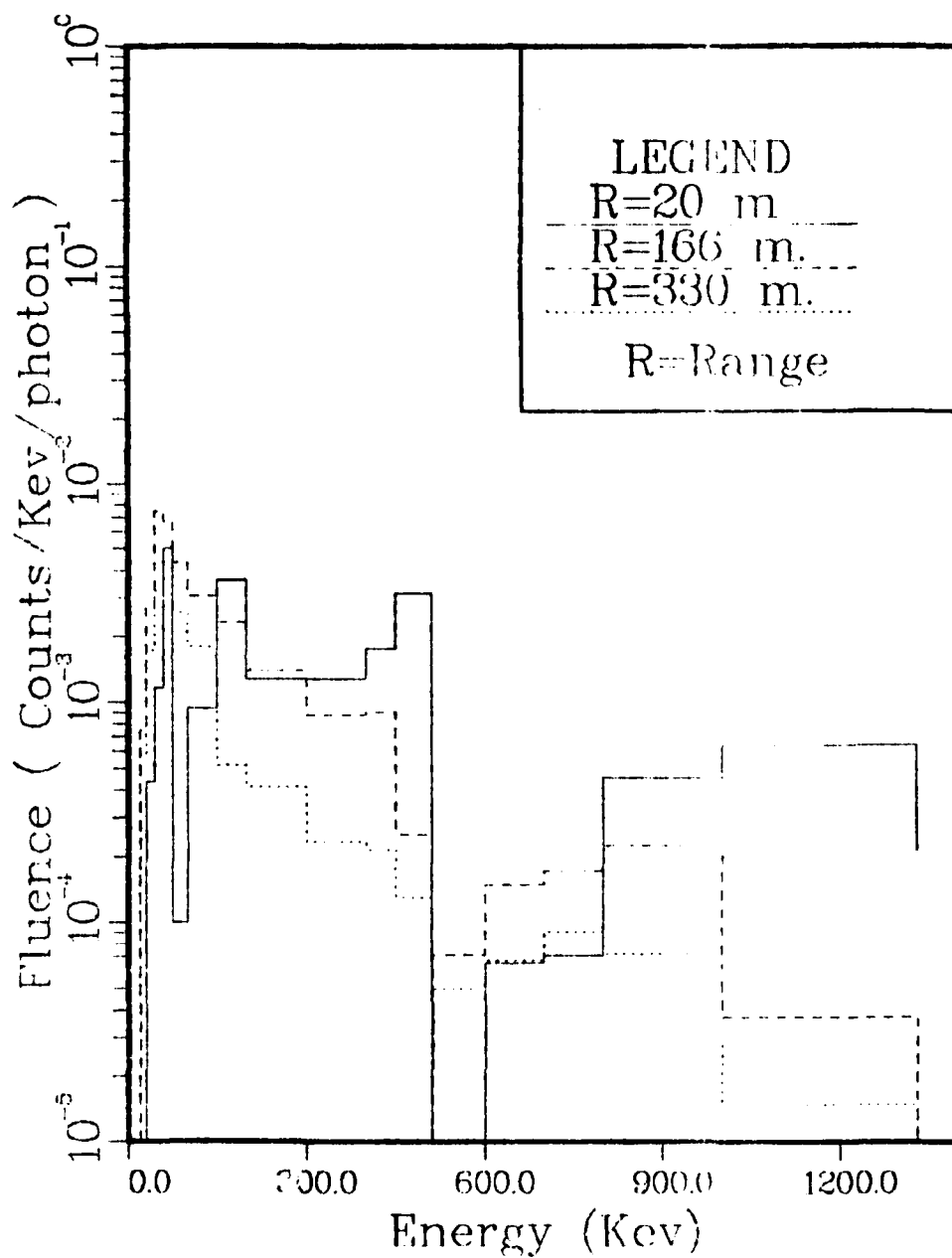


Figure 20. "Good" spectra of Source X at ranges of 20, 166, and 330 meters.

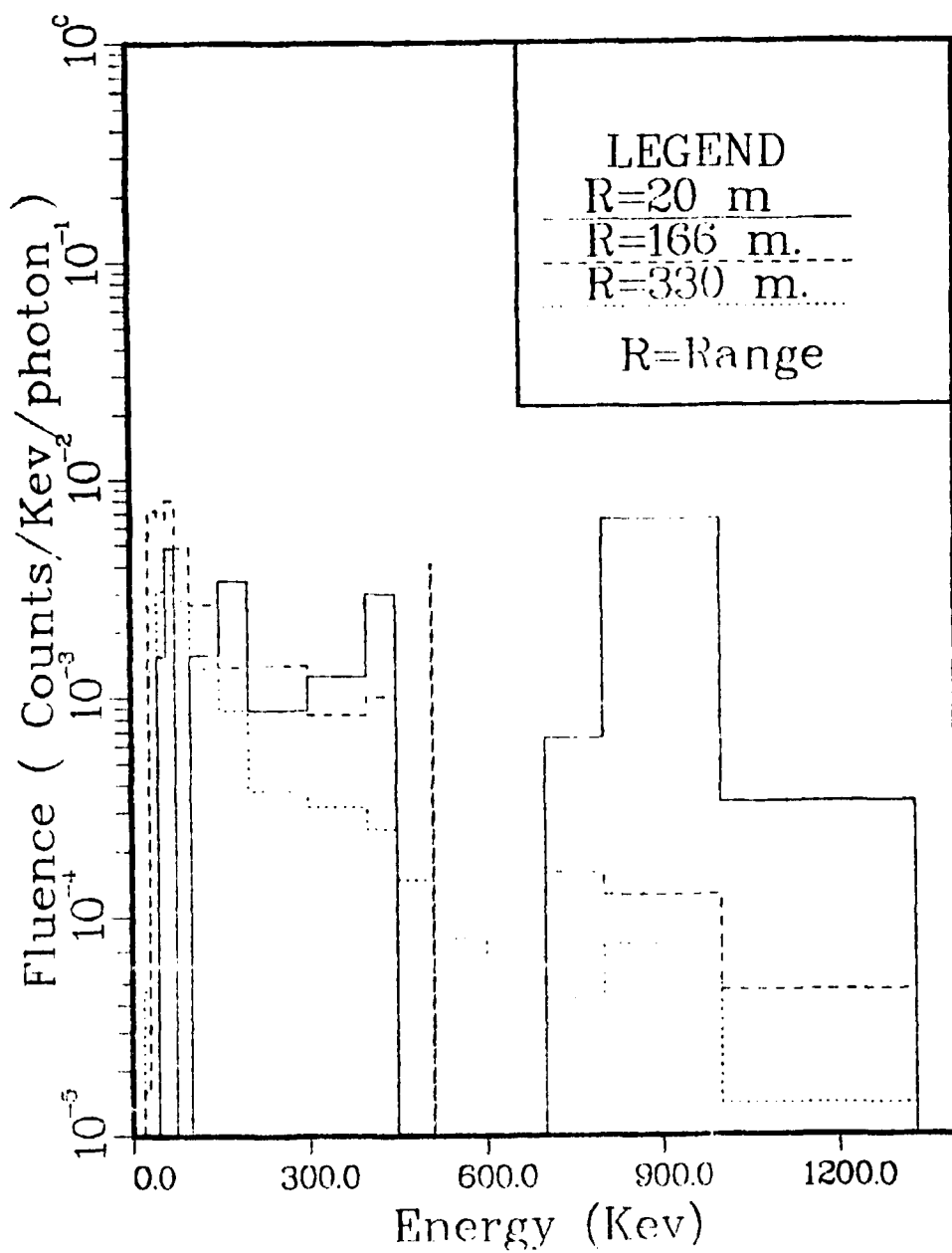


Figure 21. "Poor" spectra of Source X at ranges of 20, 166, and 330 meters.

### III. CROSS-CORRELATION

#### Background

Cross-correlation may be defined as a mathematical technique for determining the degree of similarities which exist between two spectra. With the introduction of digital computers, cross-correlation became a common means of comparing a measured spectrum with a set of library spectra in search of sought-for spectral features. The main application of cross-correlation techniques has long been to detect signals. The first discipline to widely use correlation, and probably the field that still uses it most today, is the communications field. The application of recognizing periodic communication type signals in noise was discussed as early as 1950 (Ref 5). Several books that deal with correlation techniques have been written with application primarily to communications (Refs 6;7).

Several articles were found that specifically applied a cross-correlation technique in analysis of nuclear spectra (Refs 8;9;10;11). The articles presented the application of correlation technique in isolating and enhancing structure in pulse-height spectrum data. The spectra peaks were the main structure of concern. This type of correlation method is a peak extraction method. A peak extraction method is any technique which can determine if a gaussian pulse exists within the random noise and background of a spectrum.

An article in 1973 by Horlick (Ref 12) presented typical results that are obtained when entire spectra are cross-correlated and how the resulting cross-correlation patterns can be interpreted. The spectra

used were line emission spectra for Co, Ni, and Fe. This article gave the clearest meaning to generating library spectra of known sources, making a measurement on an unknown source, and then matching the unknown spectrum with the proper library spectrum. The match then identifies the source of the unknown spectrum.

### Decision Rules

In order to analyze the unknown spectrum and make a choice as to its source, some form of a decision must be made. Two decisions rules were proposed and studied. Both of these decision rules employ cross-correlation coefficients.

Cross-Correlation Coefficient. The cross-correlation coefficient is defined here as

$$\rho_{i,j} = \sum_{k=1}^G C_k(i) * C_k(j) \quad (1)$$

where,

$\rho_{i,j}$  = cross-correlation coefficient of spectrum i correlated with spectrum j.

G = number of energy groups being used to define the correlated spectra.

$C_k(i)$  = spectrum i's fluence estimate of energy group k.

$C_k(j)$  = spectrum j's fluence estimate of energy group k.

$\rho_{i,j}$  may be normalized by the denominator,

$$\sqrt{\sum_{k=1}^G C_k(i)^2 * \sum_{k=1}^G C_k(j)^2} \quad (2)$$

such that

$$0 \leq \rho_{i,j} \leq 1$$

and for a spectrum correlated with itself

$$\rho_{i,j} = 1$$

The actual definition of (1) is quite flexible and may be seen written in different forms in other literature (Ref 6). In determining the form of the correlation equation a simple expression was desired that could be applied to the group nature of the spectra as well as be easily analyzed in a statistical fashion. Equation (1) meets these desired conditions for the coefficient equation.

The value of  $\rho_{i,j}$  can be thought of as a measure of all spectral features shared by the two spectra on a percentage basis. The closer  $\rho_{i,j}$  is to 1 the more features the two correlated spectra have in common with one another.

Largest Coefficient. In most applications of the cross-correlation coefficient, the match for a measured spectrum has depended upon cross-correlating the measured spectrum with the library spectra and choosing the largest correlation coefficient as the indication for the correct match. This method, choosing the largest coefficient, constitutes the first decision rule.

Before the second decision rule is presented it is instructive to introduce the meaning of the library matrix and the measured matrix.

The library matrix contains the coefficient values of the library spectra correlated with one another. Since three sources comprise the entire library, a matrix with dimensions of 3X3 will be formed. The

library matrix for the 3 sources looks like the following:

$$\begin{bmatrix} \rho_{LA,LA} & \rho_{LA,LB} & \rho_{LA,LC} \\ \rho_{LB,LA} & \rho_{LB,LB} & \rho_{LB,LC} \\ \rho_{LC,LA} & \rho_{LC,LB} & \rho_{LC,LC} \end{bmatrix}$$

$\rho_{LA,LB}$  would be the cross-correlation coefficient of the library spectrum A with the library spectrum B.

Each row vector of the library matrix may be viewed as the "components" of library spectrum A (row 1), library spectrum B (row 2), and library spectrum C (row 3).

Due to the nature of the correlation equation, this matrix is symmetric, i.e.  $\rho_{LA,LB} = \rho_{LB,LA}$ ,  $\rho_{LA,LC} = \rho_{LC,LA}$ , and  $\rho_{LB,LC} = \rho_{LC,LB}$ . The diagonal terms will have a value of 1, since cross-correlating a spectrum with itself must result in a perfect match (See page for normalization procedure).

These coefficients each have a standard deviation of the mean,  $\sigma_{\rho}$ , due to the nature of their generation. Note, the diagonal  $\sigma$ 's are equal to zero because the normalization procedure always gives a value of one for the diagonal coefficients.

The completely described library matrix looks like the following:

$$\begin{bmatrix} \rho_{LA,LA} \pm \sigma_{LA,LA} & \rho_{LA,LB} \pm \sigma_{LA,LB} & \rho_{LA,LC} \pm \sigma_{LA,LC} \\ \rho_{LB,LA} \pm \sigma_{LB,LA} & \rho_{LB,LB} \pm \sigma_{LB,LB} & \rho_{LB,LC} \pm \sigma_{LB,LC} \\ \rho_{LC,LA} \pm \sigma_{LC,LA} & \rho_{LC,LB} \pm \sigma_{LC,LB} & \rho_{LC,LC} \pm \sigma_{LC,LC} \end{bmatrix}$$

As with the coefficients, the standard deviations are also symmetric, i.e.  $\sigma_{LA, LB} = \sigma_{LB, LA}$ ,  $\sigma_{LB, LC} = \sigma_{LC, LB}$ , and  $\sigma_{LA, LC} = \sigma_{LC, LA}$ .

A library matrix was calculated for each distance the three sources were transported.

The measured matrix contains the coefficient values of the measured spectrum cross-correlated with the library spectra at corresponding transport distances. In this case, a matrix with dimensions  $1 \times 3$  results, giving

$$\begin{bmatrix} \rho_{M''i'', LA} & \rho_{M''i'', LB} & \rho_{M''i'', LC} \end{bmatrix}$$

where  $\rho_{M''i'', LA}$  is read the cross-correlation coefficient of measured spectrum "i" ("i" indicates the correct source for the measured spectrum) correlated to library spectrum A and so forth.

Likelihood Function. The likelihood function is a probability density that will be used for the second decision rule.

The likelihood function used in this study is given by

$$P(M=i) = \frac{\text{EXP} \left\{ -\frac{1}{2} \left[ \left( \frac{\rho_{Mi, LA} - \rho_{Li, LA}}{\sigma_{Mi, LA}} \right)^2 + \left( \frac{\rho_{Mi, LB} - \rho_{Li, LB}}{\sigma_{Mi, LB}} \right)^2 + \left( \frac{\rho_{Mi, LC} - \rho_{Li, LC}}{\sigma_{Mi, LC}} \right)^2 \right] \right\}}{\sqrt{(2\pi)^3 \sigma_{Mi, LA} \sigma_{Mi, LB} \sigma_{Mi, LC}}} \quad (1)$$

where,

$M$  = assumed source for the measured spectrum

$P(M=i)$  = value of the density function assuming  $M$ 's source is source  $i$ .

$\sigma_{Mi, Lj}$  = standard deviation of the correlation coefficient of the

assumed source, M, about the library spectrum of j (See Appendix B for derivation). "i" indicates the assumed source of the unknown spectrum.

Clearly, (1) consist of three gaussian density functions (gdf) multiplied together. The use of the gdf is based on the assumption that the normalized coefficients follow a gaussian distribution. This assumption must be kept in mind when the results of this decision rule are presented.

The likelihood function does several things that the measured coefficients (alone) do not. The function:

- (1) allows for the similarities of the library spectra with each other to be mathematically taken into account; and
- (2) takes into account any error associated with the library spectra and the assumed source.

The use of the likelihood function is a more mathematically correct method for determining the correct match through cross-correlation analysis (Ref 13).



#### IV. RESULTS AND DISCUSSION

To demonstrate the feasibility of extending spectrum identification by correlating spectra consisting of uncollided as well as collided photons, the measured spectra (including Source X) were correlated to each of the library spectra. The decision rules were then applied to the coefficients to see if the correct match was indicated.

The correct source of the measured spectra will be designated with an M, for measured, followed by the letter indicating its source, i.e. MA means the measured spectrum of source A. In addition a numerical value may follow M"i" indicating results for the "good" or "poor" measured spectra, i.e. MA1000 would be for "good" while MA100 would be for "poor" spectra. This nomenclature indicates the number of source photons used for their spectra generation.

The results of the decision rules are presented in the sequence in which the scenarios were presented in Chapter II.

##### Coefficient Matrices

Tables IV, V, and VI give the coefficient values for the library matrix, the "good" measured matrix, and the "poor" measured matrix, respectively. All the coefficients shown are normalized coefficients.

The associated standard deviations of the means for the library coefficients are tabulated in Table VII.

Library Matrix. Let us first look at the library matrix (Table VI) and understand some of the trends that are occurring. The most noticeable feature about the coefficients is that their values are becoming closer and closer together as range increases. This phenomenon is occurring

Table IV. Coefficient values of the library matrix

SOURCE			LEGEND
1.0000	.1478	.4325	
.1478	1.0000	.8530	
.4325	.8530	1.0000	
RANGE = 20 meters			
1.0000	.3814	.4556	
.3814	1.0000	.8587	
.4556	.8587	1.0000	
RANGE = 41 meters			
1.0000	.5031	.5727	
.5031	1.0000	.8821	
.5727	.8821	1.0000	
RANGE = 83 meters			
1.0000	.7079	.7578	
.7079	1.0000	.9174	
.7578	.9174	1.0000	
RANGE = 124.5 meters			
1.0000	.8188	.8591	
.8188	1.0000	.9469	
.8591	.9469	1.0000	
RANGE = 166. meters			
1.0000	.8735	.9163	
.8735	1.0000	.9576	
.9163	.9576	1.0000	
RANGE = 246. meters			
1.0000	.9090	.9547	
.9090	1.0000	.9626	
.9547	.9626	1.0000	
RANGE = 330. meters			
1.0000	.9138	.9696	
.9138	1.0000	.9618	
.9696	.9618	1.0000	
RANGE = 390. meters			
1.0000	.9307	.9810	
.9307	1.0000	.9559	
.9810	.9559	1.0000	

$$\begin{bmatrix} \rho_{LA,LA} & \rho_{LA,LB} & \rho_{LA,LC} \\ \rho_{LB,LA} & \rho_{LB,LB} & \rho_{LB,LC} \\ \rho_{LC,LA} & \rho_{LC,LB} & \rho_{LC,LC} \end{bmatrix}$$

Table V. Coefficients of "Good" matrix

<u>RANGE</u> <u>(meters)</u>	<u>MA1000</u>	<u>MB1000</u>	<u>MC1000</u>
20.0	(.995,.422,.497)	(.379,.999,.853)	(.470,.859,.998)
41.0	(.995,.472,.539)	(.496,.999,.883)	(.592,.860,.997)
83.0	(.990,.637,.709)	(.704,.997,.921)	(.751,.904,.996)
124.5	(.986,.759,.804)	(.818,.997,.942)	(.853,.961,.997)
166.0	(.980,.848,.893)	(.869,.992,.949)	(.904,.939,.988)
246.0	(.985,.884,.939)	(.901,.984,.918)	(.943,.950,.982)
330.0	(.953,.928,.950)	(.924,.990,.963)	(.965,.951,.977)
390.0	(.965,.916,.954)	(.921,.974,.939)	(.964,.948,.990)

Table VI. Coefficients of "Poor" matrix

<u>RANGE</u> <u>(meters)</u>	<u>MA100</u>	<u>MB100</u>	<u>MC100</u>
20.0	(.969,.500,.533)	(.364,.997,.861)	(.446,.836,.981)
41.0	(.971,.549,.600)	(.490,.941,.866)	(.530,.927,.985)
83.0	(.886,.753,.759)	(.678,.967,.866)	(.720,.898,.944)
124.5	(.907,.687,.735)	(.797,.960,.897)	(.710,.807,.831)
166.0	(.909,.715,.763)	(.795,.940,.897)	(.789,.780,.878)
246.0	(.859,.778,.842)	(.722,.841,.789)	(.919,.913,.944)
330.0	(.946,.907,.932)	(.844,.961,.914)	(.731,.696,.711)

Table VII. Standard deviations of the means for the coefficient library matrix

LIBRARY	SIGMA	RANGE = 20 meters
A	B	C
0.0	8.525E-003	7.873E-003
8.525E-003	0.0	8.101E-003
7.873E-003	8.101E-003	0.0

LIBRARY	SIGMA	RANGE = 41 meters
A	B	C
0.0	1.013E-002	1.002E-002
1.013E-002	0.0	1.133E-002
1.002E-002	1.133E-002	0.0

LIBRARY	SIGMA	RANGE = 83 meters
A	B	C
0.0	1.262E-002	1.336E-002
1.262E-002	0.0	1.268E-002
1.336E-002	1.268E-002	0.0

LIBRARY	SIGMA	RANGE = 124.5 meters
A	B	C
0.0	1.940E-002	1.800E-002
1.940E-002	0.0	1.957E-002
1.800E-002	1.957E-002	0.0

LIBRARY	SIGMA	RANGE = 160. meters
A	B	C
0.0	1.893E-002	1.893E-002
1.893E-002	0.0	1.865E-002
1.893E-002	1.865E-002	0.0

LIBRARY	SIGMA	RANGE = 246. meters
A	B	C
0.0	2.315E-002	2.414E-002
2.315E-002	0.0	2.147E-002
2.414E-002	2.147E-002	0.0

LIBRARY	SIGMA	RANGE = 330. meters
A	B	C
0.0	2.769E-002	2.863E-002
2.769E-002	0.0	3.251E-002
2.863E-002	3.251E-002	0.0

LIBRARY	SIGMA	RANGE = 390. meters
A	B	C
0.0	3.371E-002	3.166E-002
3.371E-002	0.0	2.960E-002
3.166E-002	2.960E-002	0.0

LEGEND

$\sigma_{LA,LA}$	$\sigma_{LA,LB}$	$\sigma_{LA,LC}$
$\sigma_{LB,LA}$	$\sigma_{LB,LB}$	$\sigma_{LB,LC}$
$\sigma_{LC,LA}$	$\sigma_{LC,LB}$	$\sigma_{LC,LC}$

due to the increased downscattering of the source photons as range increases. The downscattering photons cause the group bins of the lower energies to fill up creating a higher correlation between the library spectra due to the spectral differences disappearing. This downscatter is evident in any of the spectra figures.

Measured Matrix. Tables V and VI give the measured matrix for the "good" and "poor" spectra, respectively.

Like the library coefficients the measured coefficients are becoming closer and closer as range increases. Of course, this was expected since the same sources are being used and the spectral features of these measured spectra are disappearing as well.

#### Library vs. Measured Coefficients

The purpose of generating coefficients is to compare the library coefficients with the measured coefficients, apply the decision rules, and study the indicated match. The measured coefficients are plotted along with the correct row of the library coefficients in Figures 22 through 27 (Source X will be presented later).

The open symbols, connected by some line "type", are coefficients of the indicated library spectrum correlated with itself and the other library spectra. The crossed-in symbols are the coefficients of the measured spectrum (unknown source) with the library spectra. If the measured spectrum's source is the same assumed library source, then each paired symbol, one open and one crossed, should be "close" to one another or show the same ordering at each range. These figures indicate that these trends are true.

These figures show several things. First, the "good" spectra

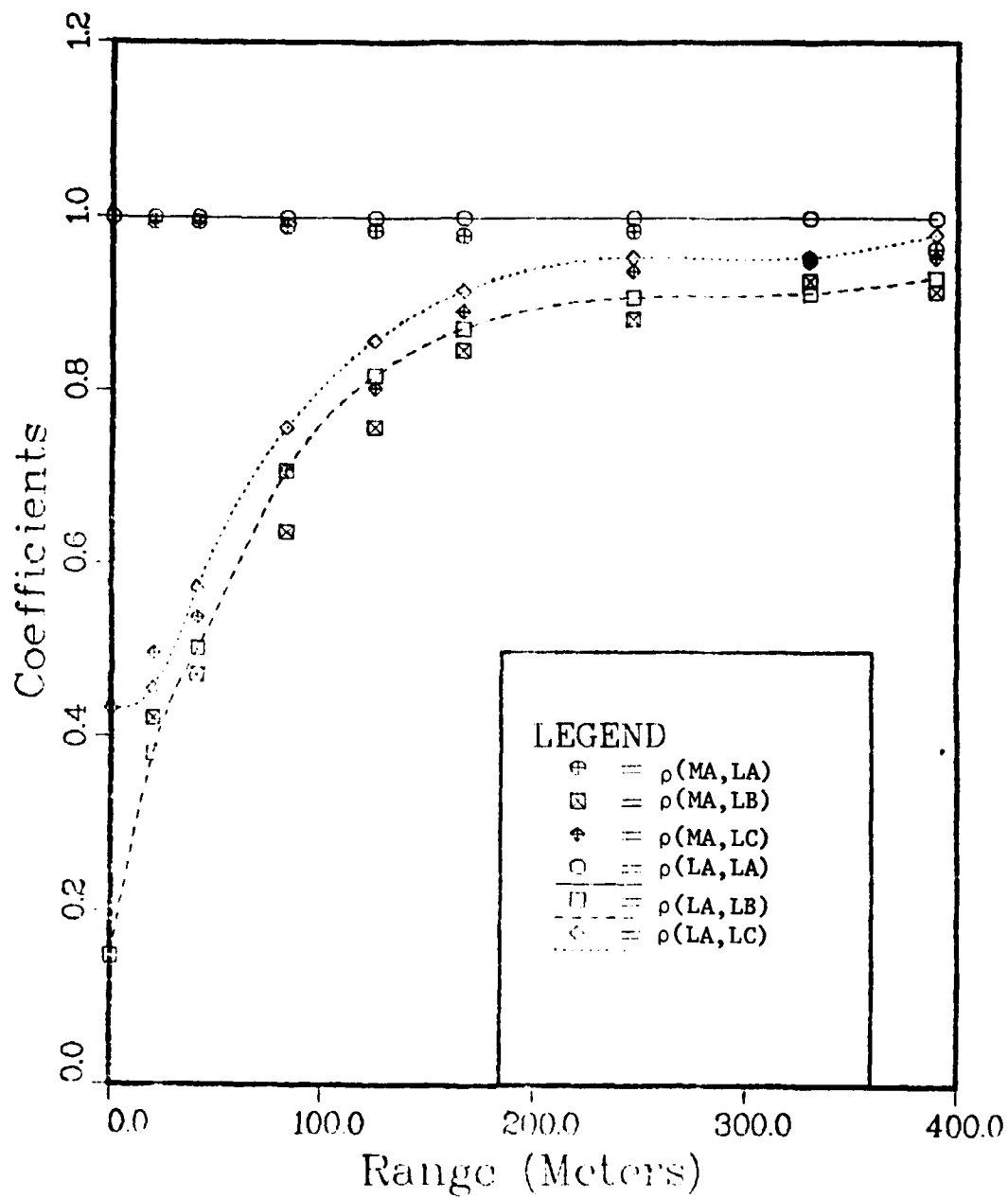


Figure 22. Measured matrix, MA (unknown: Source A) vs library coefficients of row 1.

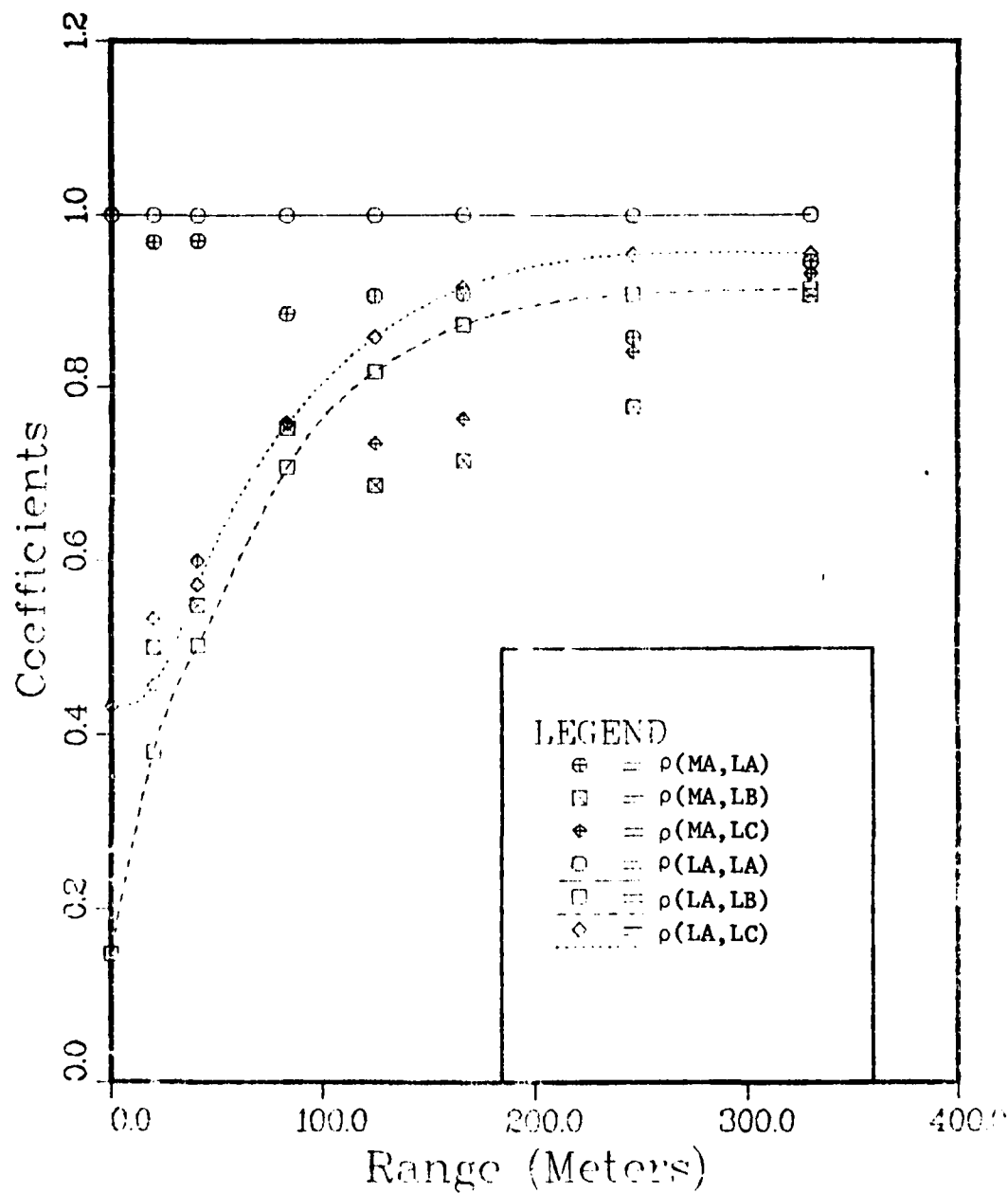


Figure 23. Measured matrix, MA100 (unknown: Source A) vs library coefficients of row 1.

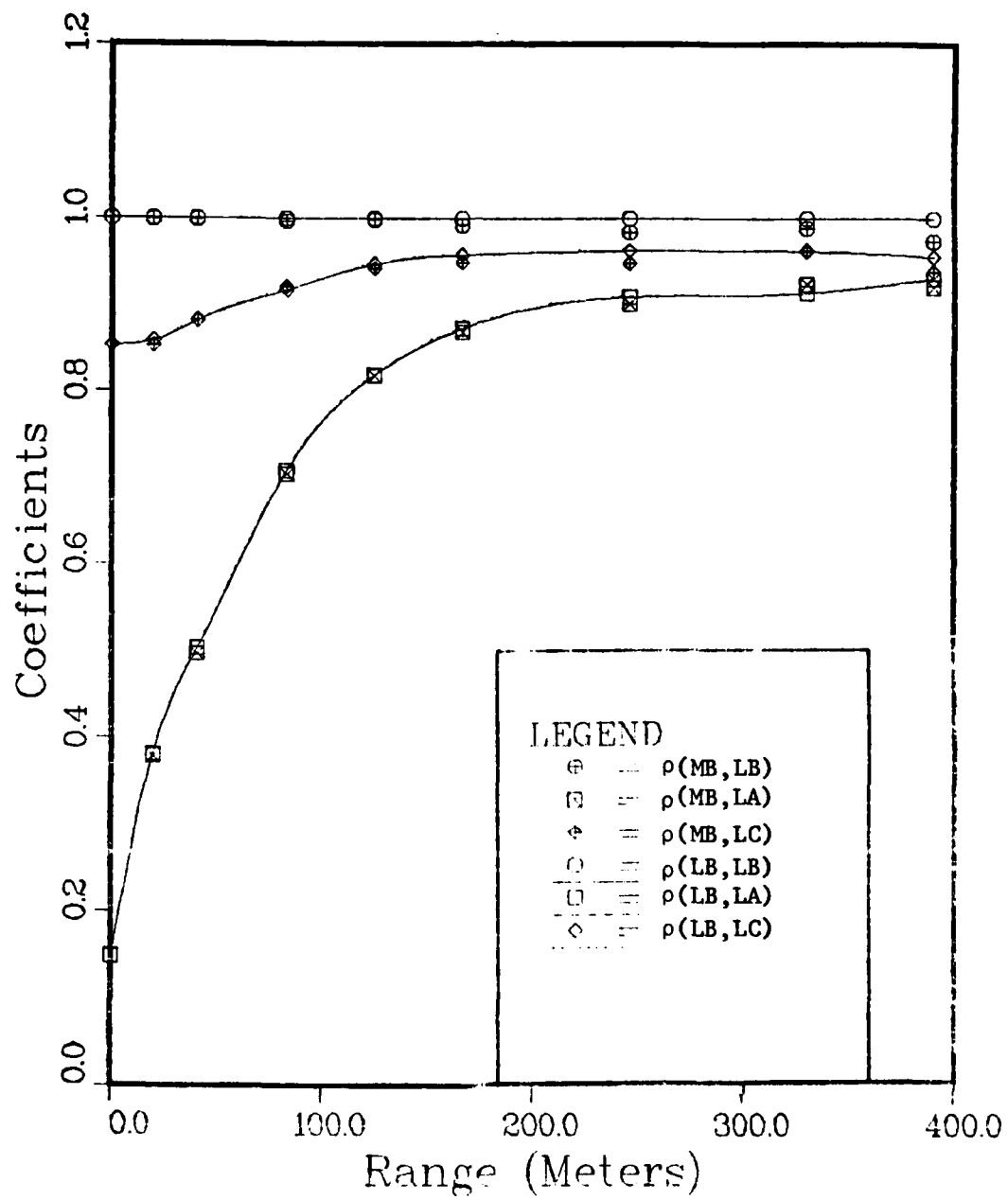


Figure 24. Measured matrix, MB (unknown: Source B) vs library coefficients of row 2.



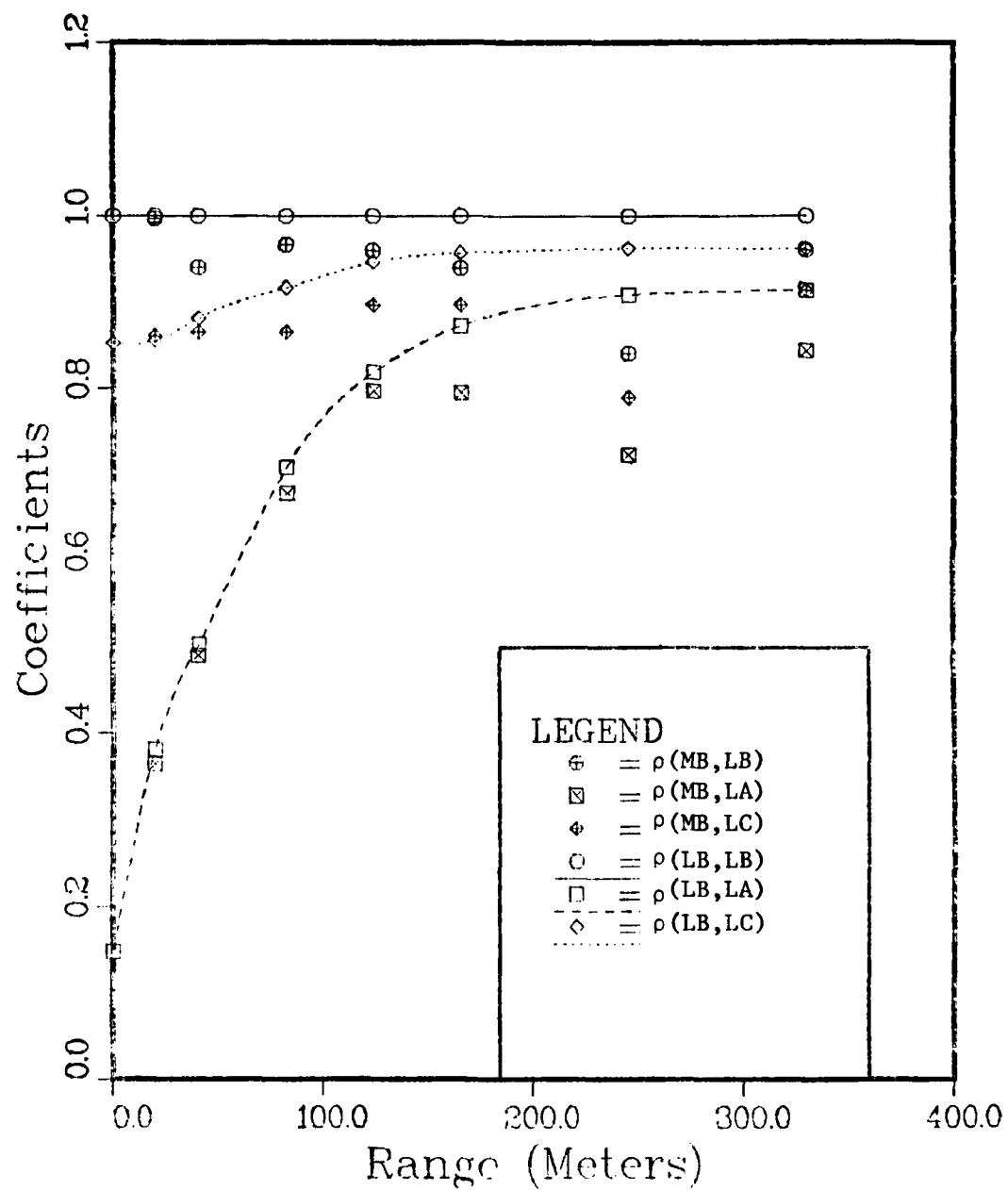


Figure 25. Measured matrix, MB100 (unknown: Source B) vs library coefficients of row 2.

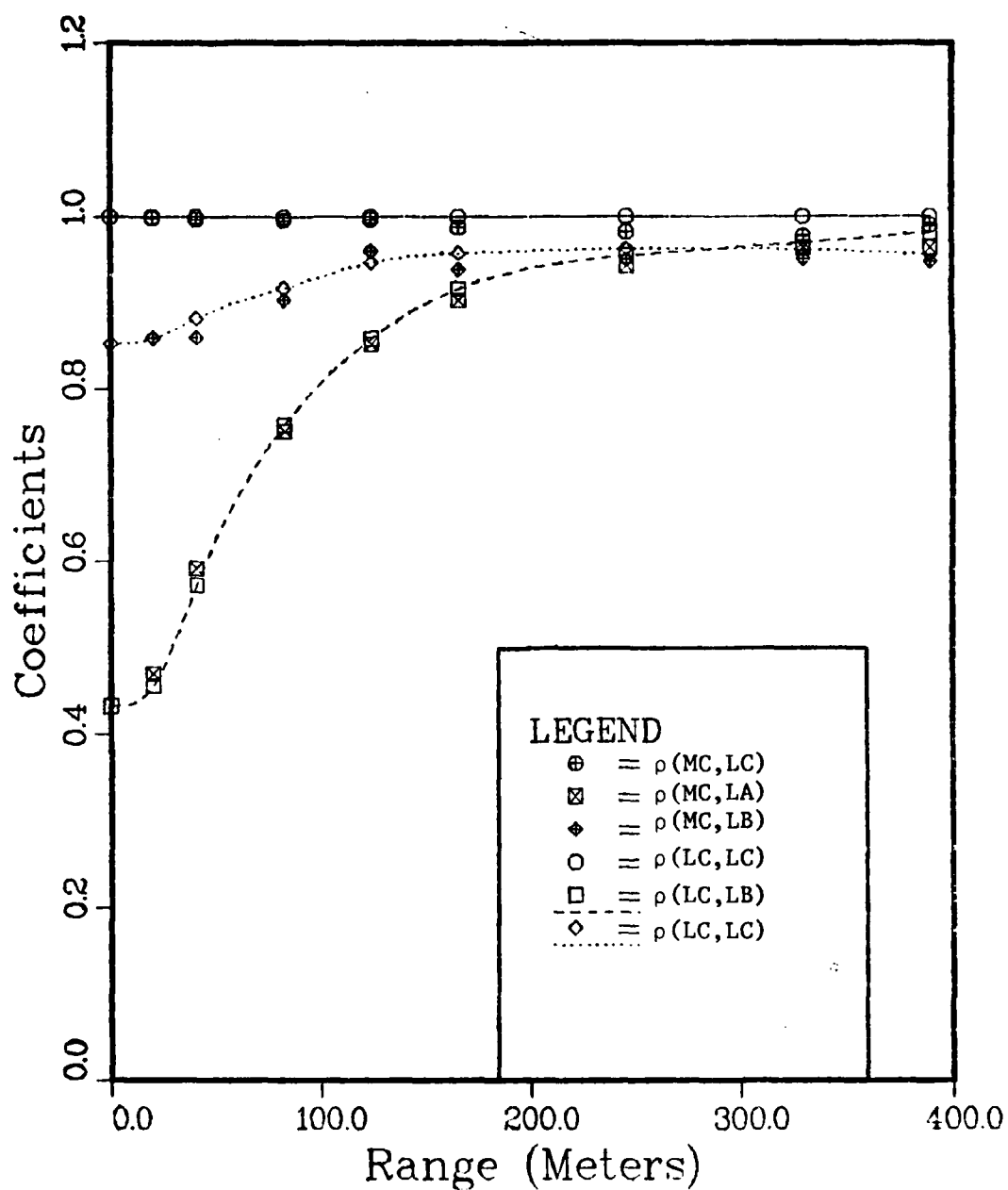


Figure 26. Measured matrix, MC (unknown: Source C) vs library coefficients of row 3.

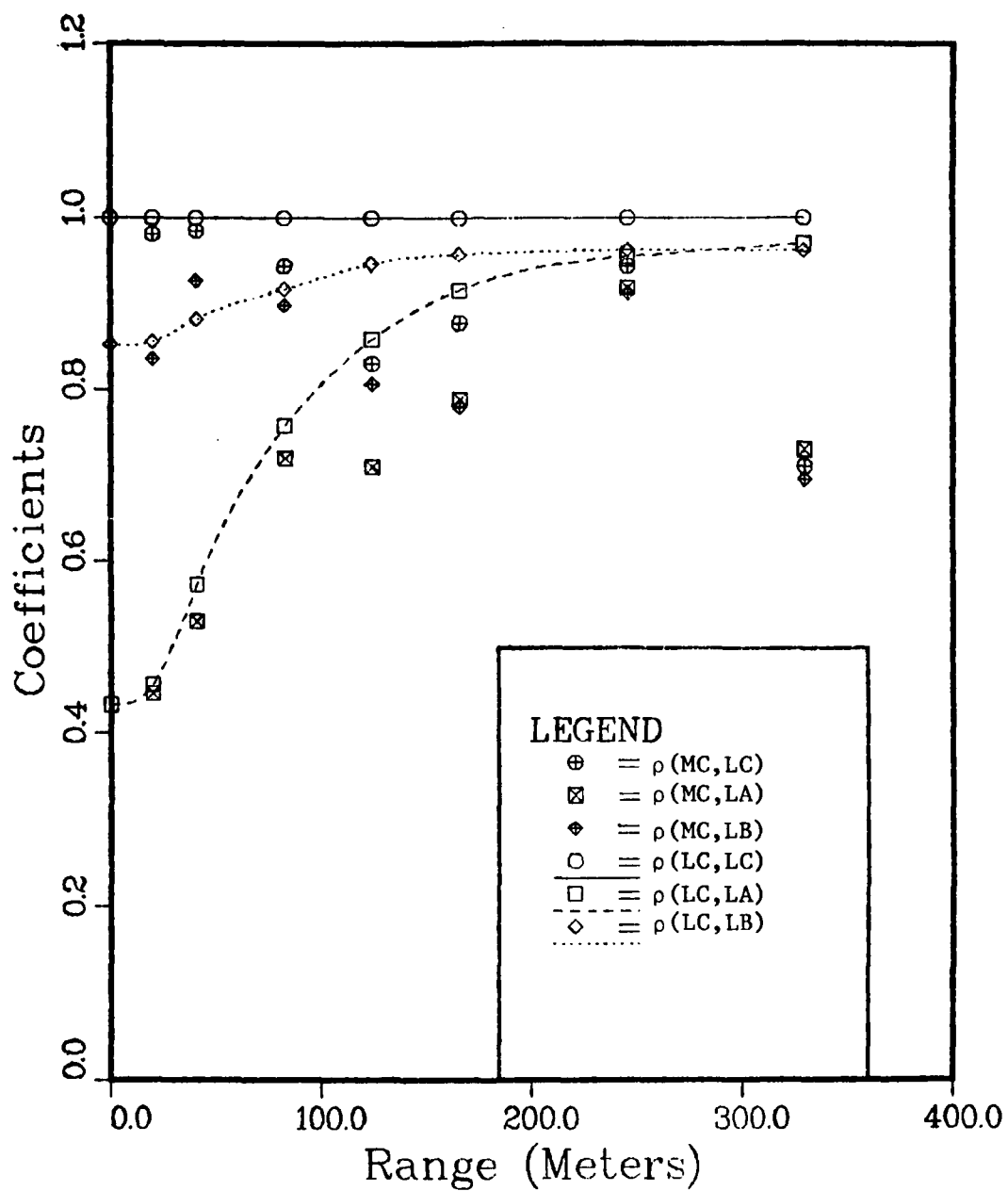


Figure 27. Measured matrix, MC100 (unknown: Source C) vs library coefficients of row 3.

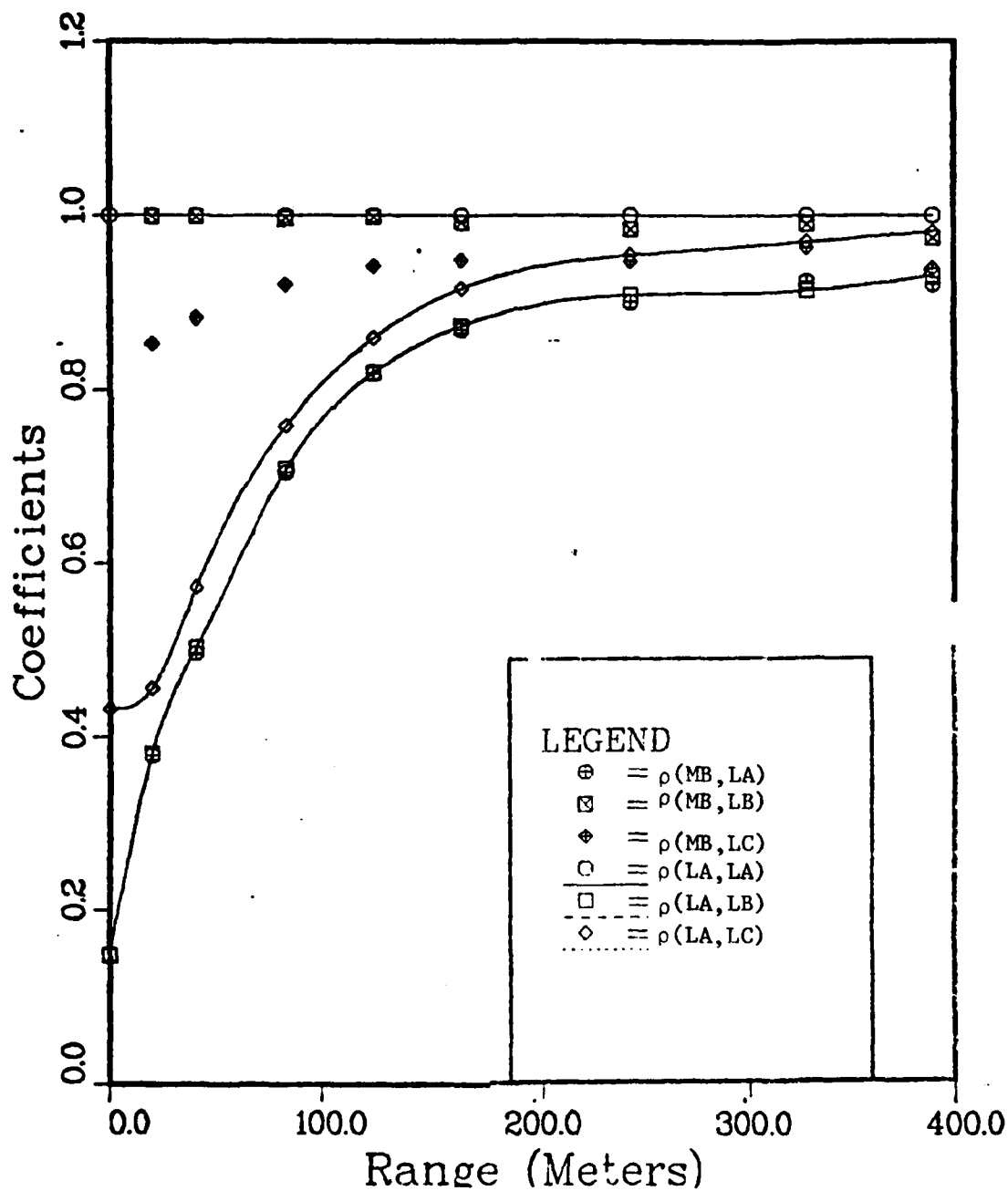


Fig. 28. Measured matrix, MB (unknown: Source B) vs. library coefficients of row 1 (THE SOURCE A ROW IN THE LIBRARY MATRIX)

coefficients, at each range, "closer" than the "poor" spectra coefficients. This difference is due to the "poor" fluences being randomly distributed about the means of the library with greater deviation than the "good" spectra. This distribution of the "good" vs "poor" has already been mentioned in a previous section. Here the plots illustrate the difference between the distributions.

Another feature about the coefficients is that as range increases the coefficients begin to converge on one another making it difficult to tell the coefficients apart. This bunching implies that identifying the source of the measured spectra will also become more difficult as range increases and spectral features disappear.

Figure 28 shows what happens when the measured matrix coefficients are compared to the wrong row of the library. In other words the wrong source. The symbols are no longer paired and the largest measured coefficient is  $\rho_{MB,LB}$ , not  $\rho_{MB,LA}$ .  $\rho_{MB,LB}$  is the coefficient corresponding to the assumed Source B (wrong in this case).

#### Results of the Decision Rules

So far the relationship between the library and measured coefficients have been shown. The decision rules are now applied to the coefficients in order to study how the correct match is affected as range increases. The maximum studied range at which the correct match was chosen will then be used to compare the photopeak identification method to these proposed methods.

Largest Coefficient. The largest coefficient in the measured matrix will determine the most likely source for the measured spectrum. This decision rule has been used in previous work (Ref 12). By looking

at Tables V and VI as well as Figures 22-27 it can be seen that the largest coefficient chose the correct source (excluding source X) in every case but for the MC100 matrix at the range of 330 meters (See Figure 27).

By employing this decision rule the correct source is chosen in every case except for the "poor" spectrum of source C at the 330 meter range. These results show that this decision rule has a great deal of merit for identifying unknown sources. What is even more encouraging about these results is that even with "bad statistics" the proper choice can be indicated. ("Bad statistics" is only a relative term, but an indication of the success that can be obtained with this decision rule has been demonstrated.)

In all these cases only one out of the thirty-eight matches indicated the wrong source for the measured spectra.

Likelihood Function. The second decision rule utilizes the likelihood function. In an analogous fashion as the first decision rule, the rule would state that the largest value for the likelihood function indicates the correct match.

Recall the likelihood function is given by

$$P(M=i) = \frac{\text{EXP} \left\{ -\frac{1}{2} \left[ \left( \frac{\rho_{Mi,LA} - \rho_{Li,LA}}{\sigma_{Mi,LA}} \right)^2 + \left( \frac{\rho_{Mi,LB} - \rho_{Li,LB}}{\sigma_{Mi,LB}} \right)^2 + \left( \frac{\rho_{Mi,LC} - \rho_{Li,LC}}{\sigma_{Mi,LC}} \right)^2 \right] \right\}}{\sqrt{(2\pi)^3 \sigma_{Mi,LA} \sigma_{Mi,LB} \sigma_{Mi,LC}}}$$

Table VIII gives the value of the likelihood function along with the maximum the function could possibly be. The maximum would indicate that all coefficient pairs were perfectly matched, i.e. each pair is exactly equal to each other, and therefore, the numerator would reduce to one.

By noting the maximum indicated for each assumed source, it can be seen that the values of the likelihood functions are weighted differently (due to the process used in determining the standard deviations used in the function). If the function is divided by its maximum value, then a percentage will be found that indicates the degree the function is to a perfect match. This ratio will be referred to as the normalized likelihood function. The largest normalized likelihood function is used as the second decision rule. Table IX gives the results.

Table IX indicates the wrong matches in the location where the symbol "+" is present.

#### Largest Coefficient vs. Likelihood Function

From the previous two sections, the identification of the correct source, by either decision rule, gives comparable results. While the rule utilizing the likelihood function indicated three wrong matches compared to one wrong match for the coefficient rule, each wrong match was indicated only for the "poor" spectra. In general terms, the two rules must be considered to be equal in their success for choosing the correct source.

#### Cross-Correlation vs. Photopeak Identification

In order to make the comparison between the cross-correlation

Table VIII

(a) Likelihood function values for measured source MA1000.

<u>RANGE</u> <u>meter</u>	<u>ASSUMED</u> <u>SOURCE</u>	<u>P</u> ( <u>ASSUMED</u> ) <u>SOURCE</u>	<u>P<sub>max</sub></u> ( <u>ASSUMED</u> ) <u>SOURCE</u>	<u>P</u> <u>P<sub>max</sub></u>
20	A	1.4E+2	3.6E+2	.374
20	B	2.5E-369	4.5E+3	0.
20	C	1.5E-269	2.7E+3	0.
41	A	4.2E+2	6.6E+2	.632
41	B	2.5E-125	1.4E+3	0.
41	C	7.3E-78	9.5E+2	0.
83	A	1.2E+2	3.2E+2	.369
83	B	1.8E-36	1.2E+3	0.
83	C	8.4E-15	4.2E+2	0.
124.5	A	7.8E+1	1.3E+2	.608
124.5	B	2.2E-2	1.4E+2	0.
124.5	C	1.6E-2	2.1E+2	0.
166.0	A	9.5E+1	1.2E+2	.789
166.0	B	2.9E+1	1.9E+2	.237
166.0	C	6.9E+1	1.7E+2	.403
246.0	A	5.6E+1	5.9E+1	.952
246.0	B	2.8E+1	1.6E+2	.177
246.0	C	6.5E+0	8.7E+0	.747
330.0	A	4.2E+1	4.6E+1	.900
330.0	B	2.8E+1	3.5E+1	.811
330.0	C	2.9E+1	3.2E+1	.888
390.0	A	2.0E+1	2.1E+1	.948
390.0	B	2.7E+1	3.5E+1	.776
390.0	C	4.3E+1	5.1E+1	.845



Table VIII

(b) Likelihood function values for measured source MB1000.

<u>RANGE</u> <u>meter</u>	<u>ASSUMED</u> <u>SOURCE</u>	<u>P</u> <u>(ASSUMED)</u> <u>(SOURCE)</u>	<u>P</u> <u>(ASSUMED)</u> <u>Pmax (SOURCE)</u>	<u>P</u> <u>Pmax</u>
20	A	1.6E-71	3.6E+2	0.
20	B	4.4E+3	4.5E+3	.973
20	C	5.6E-8	2.7E+3	0.
41	A	6.6E+2	6.6E+2	0.
41	B	1.4E+3	1.4E+3	.964
41	C	9.5E+2	9.5E+2	0.
83	A	3.2E+2	3.2E+2	0.
83	B	1.2E+3	1.2E+3	.983
83	C	4.3E+2	4.2E+2	.71
124.5	A	1.3E+2	1.3E+2	0.
124.5	B	1.4E+2	1.4E+2	.993
124.5	C	2.1E+2	2.1E+2	.444
166.0	A	9.5E+0	1.2E+2	.078
166.0	B	1.9E+2	1.9E+2	.984
166.0	C	9.4E+1	1.7E+2	.548
246.0	A	2.6E+1	5.9E+1	.436
246.0	B	1.3E+2	1.6E+2	.8141
246.0	C	4.7E+0	8.7E+0	.541
330.0	A	2.9E+1	4.6E+1	.636
330.0	B	3.5E+1	3.5E+1	.994
330.0	C	2.8E+1	3.2E+1	.869
390.0	A	1.7E+1	2.1E+1	.791
390.0	B	3.4E+1	3.5E+1	.968
390.0	C	3.6E+1	5.1E+1	.708

Table VIII

(c) Likelihood function values for measured source MC1000.

<u>RANGE</u> <u>meter</u>	<u>ASSUMED</u> <u>SOURCE</u>	<u>P (ASSUMED)</u> <u>P (SOURCE)</u>	<u>Pmax (ASSUMED)</u> <u>Pmax (SOURCE)</u>	<u>P</u> <u>Pmax</u>
20	A	4.0E-71	3.6E+2	0.
20	B	3.3E-13	4.5E+3	0.
20	C	2.0E+3	2.7E+3	.752
41	A	1.8E-48	6.6E+2	0.
41	B	3.7E-4	1.4E+3	0.
41	C	6.9E+2	9.5E+2	.729
83	A	1.5E-8	3.2E+2	0.
83	B	5.3E+0	1.2E+3	.005
83	C	4.2E+2	4.2E+2	.963
124.5	A	9.2E-1	1.3E+2	0.
124.5	B	9.0E+1	1.4E+2	.645
124.5	C	2.0E+2	2.1E+2	.981
166.0	A	2.7E+1	1.2E+2	.226
166.0	B	1.1E+2	1.9E+2	.580
166.0	C	1.6E+2	1.7E+2	.936
246.0	A	4.4E+1	5.9E+1	.755
246.0	B	1.1E+2	1.6E+2	.705
246.0	C	8.5E+0	8.7E+0	.971
330.0	A	4.1E+1	4.6E+1	.898
330.0	B	2.9E+1	3.5E+1	.834
330.0	C	3.4E+1	3.2E+1	.970
390.0	A	2.0E+1	2.1E+1	.957
390.0	B	3.0E+1	3.5E+1	.853
390.0	C	5.0E+1	5.1E+1	.978

Table VIII

(d) Likelihood function values for measured source MA100.

<u>RANGE</u> <u>meter</u>	<u>ASSUMED</u> <u>SOURCE</u>	<u>P</u> <sup>(ASSUMED)</sup> <u>(SOURCE)</u>	<u>P<sub>max</sub></u> <sup>(ASSUMED)</sup> <u>(SOURCE)</u>	<u>P</u> <u>P<sub>max</sub></u>
20	A	1.8E+1	1.1E+2	.159
20	B	5.8E-111	8.8E+2	0.
20	C	4.8E-72	5.1E+2	0.
41	A	1.4E+2	2.1E+2	.643
41	B	1.5E-34	2.7E+2	0.
41	C	8.0E-20	1.8E+2	0.
83	A	1.0E+1	1.0E+2	.097
83	B	3.2E-4	2.2E+2	0.
83	C	3.0E-1	8.0E+1	0.
124.5	A	8.3E+0	4.1E+1	.202
124.5	B	6.9E-1	2.6E+1	.027
124.5	C	4.4E-1	3.9E+1	.011
166.0	A	6.1E+0	3.9E+1	.159
166.0	B	9.1E-1	3.6E+1	.026
166.0	C	1.1E+0	3.2E+1	.035
246.0	A	4.3E+0	1.9E+1	.230
246.0	B	4.8E+0	2.9E+1	.165
246.0	C	4.3E+0	1.6E+1	.266
330.0	A	1.3E+1	1.5E+1	.879
330.0	B	5.6E+0	6.5E+0	.865
330.0	C	5.4E+0	5.9E+0	.922

Table VIII

(e) Likelihood function values for measured source MB100.

<u>RANGE</u> <u>meter</u>	<u>ASSUMED</u> <u>SOURCE</u>	<u>P</u> (ASSUMED) <u>(SOURCE)</u>	<u>P<sub>max</sub></u> (ASSUMED) <u>(SOURCE)</u>	<u>P</u> <u>P<sub>max</sub></u>
20	A	6.0E-28	1.1E+2	0.
20	B	7.5E+2	8.8E+2	.847
20	C	6.5E-2	5.1E+2	0.
41	A	1.3E-40	2.1E+2	0.
41	B	1.8E+2	2.7E+2	.695
41	C	1.0 E+1	1.8E+2	.058
83	A	6.7E-8	1.0E+2	0.
83	B	1.3E+2	2.2E+2	.599
83	C	1.7E+1	8.0E+1	.212
124.5	A	4.8E-1	4.1E+1	.012
124.5	B	2.3E+1	2.6E+1	.891
124.5	C	2.3E+1	3.9E+1	.585
166.0	A	9.5E-1	3.9E+1	.024
166.0	B	2.2E+1	3.6E+1	.617
166.0	C	1.3E+1	3.2E+1	.422
246.0	A	2.0E-1	1.9E+1	.011
246.0	B	1.8E+0	2.9E+1	.063
246.0	C	1.6E+1	1.6E+1	.010
330.0	A	5.8E+0	1.5E+1	.389
330.0	B	5.8E+0	6.5E+0	.898
330.0	C	4.5E+0	5.9E+0	.772

Table VIII

(f) Likelihood function values for measured source MC100.

<u>RANGE</u> <u>meter</u>	<u>ASSUMED</u> <u>SOURCE</u>	<u>P (ASSUMED)</u> <u>P (SOURCE)</u>	<u>P<sub>max</sub> (ASSUMED)</u> <u>P<sub>max</sub> (SOURCE)</u>	<u>P</u> <u>P<sub>max</sub></u>
20	A	1.1E-24	1.1E+2	0.
20	B	1.4E-2	8.8E+2	0.
20	C	4.3E+2	5.1E+2	.854
41	A	9.2E-38	2.1E+2	0.
41	B	5.0E+1	2.7E+2	.187
41	C	1.1E+2	1.8E+2	.608
83	A	6.4E-6	1.0E+2	0.
83	B	7.8E+1	2.2E+2	.350
83	C	6.0E+1	8.0E+1	.745
124.5	A	1.2E-2	4.1E+1	.003
124.5	B	5.1E+0	2.6E+1	.201
124.5	C	3.0E+0	3.9E+1	.077
166.0	A	6.8E-1	3.9E+1	.017
166.0	B	5.2E+0	3.6E+1	.146
166.0	C	4.6E+1	3.2E+1	.147
246.0	A	1.3E+1	1.9E+1	.706
246.0	B	2.4E+1	2.9E+1	.810
246.0	C	1.4E+1	1.6E+1	.875
330.0	A	2.3E-1	1.5E+1	.015
330.0	B	1.4E+0	6.5E+0	.219
330.0	C	6.8E-1	5.9E+0	.116

Table IX. Results of "matching" using  
the likelihood function

Range (meter)	MA 1000	MA 100	MB 1000	MB 100	MC 1000	MC 100
20						
41						
83						
124.5						
166.0						
245.6		+				
330.		+				+
390.		N/A		N/A		N/A

+ Wrong match indicated.

technique and the photopeak technique, the range where photopeak identification can no longer be utilized must be determined. Some subjective decision had to be made based on the spectra generated. By looking at the "good" and "poor" spectra, the range of 166 meters shows that most of the initial source structure has vanished. A way to interpret the range limit for photopeak identification is to picture the measured spectra of Source A and Source C (Figures 6 & 8) and their Source Spectra (Source B is excluded because of its lower spectra bins, i.e. easily identified from the other spectra by its upper bin limit). At this range it is becoming difficult to look at the spectra and identify the correct source. Therefore, the 166 meter range will be considered the maximum range limit where photopeak identification may be carried out successfully.

In order to help justify this range, a look at the mean free paths for a couple of the primary energy groups .8-1.MeV and .15-.2MeV translated into 1.5 and 2.8 mean free paths, respectively. This distance does not seem to be a bad choice, since the probability of the photons scattering is 77.3% and 93.9%, respectively.

The decision rule results indicate a rather large range extension for identification. This extension is by as much as twice the photopeak range limit, i.e. 166 meters compared to as high as 390 meters.

#### Source X: The Non-Library Source

Table X shows the measured coefficients of measured spectra X and X100 with the library matrix. These coefficients along with library coefficients are plotted in Figures 29 through 34.

Clearly each decision rule will always indicate the wrong match.

Table X. Coefficients of "Good" and "Poor"  
matrix of source X

<u>RANGE</u> <u>(meters)</u>	<u>MX1000</u>	<u>MX100</u>
20.0	(.431, .624, .649)	(.238, .164, .557)
41.0	(.598, .669, .701)	(.536, .594, .557)
83.0	(.814, .805, .853)	(.750, .860, .903)
124.5	(.899, .848, .890)	(.888, .784, .844)
166.0	(.954, .895, .934)	(.898, .845, .868)
246.0	(.957, .938, .967)	(.916, .839, .894)
330.0	(.970, .935, .973)	(.976, .923, .977)
390.0	(.965, .916, .954)	N/A



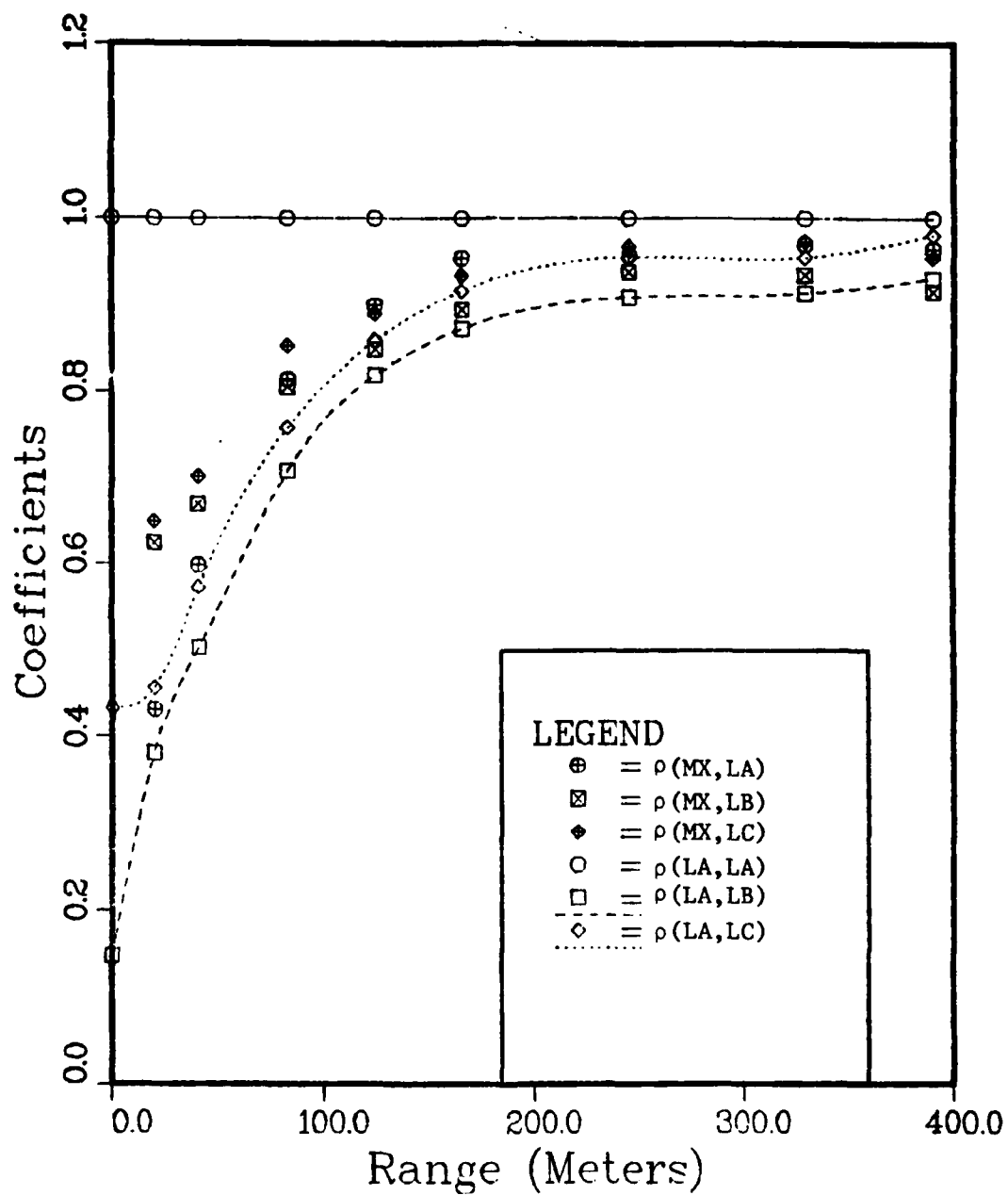


Figure 29. Measured matrix, MX (unknown: Source X) vs library coefficients of row 1.

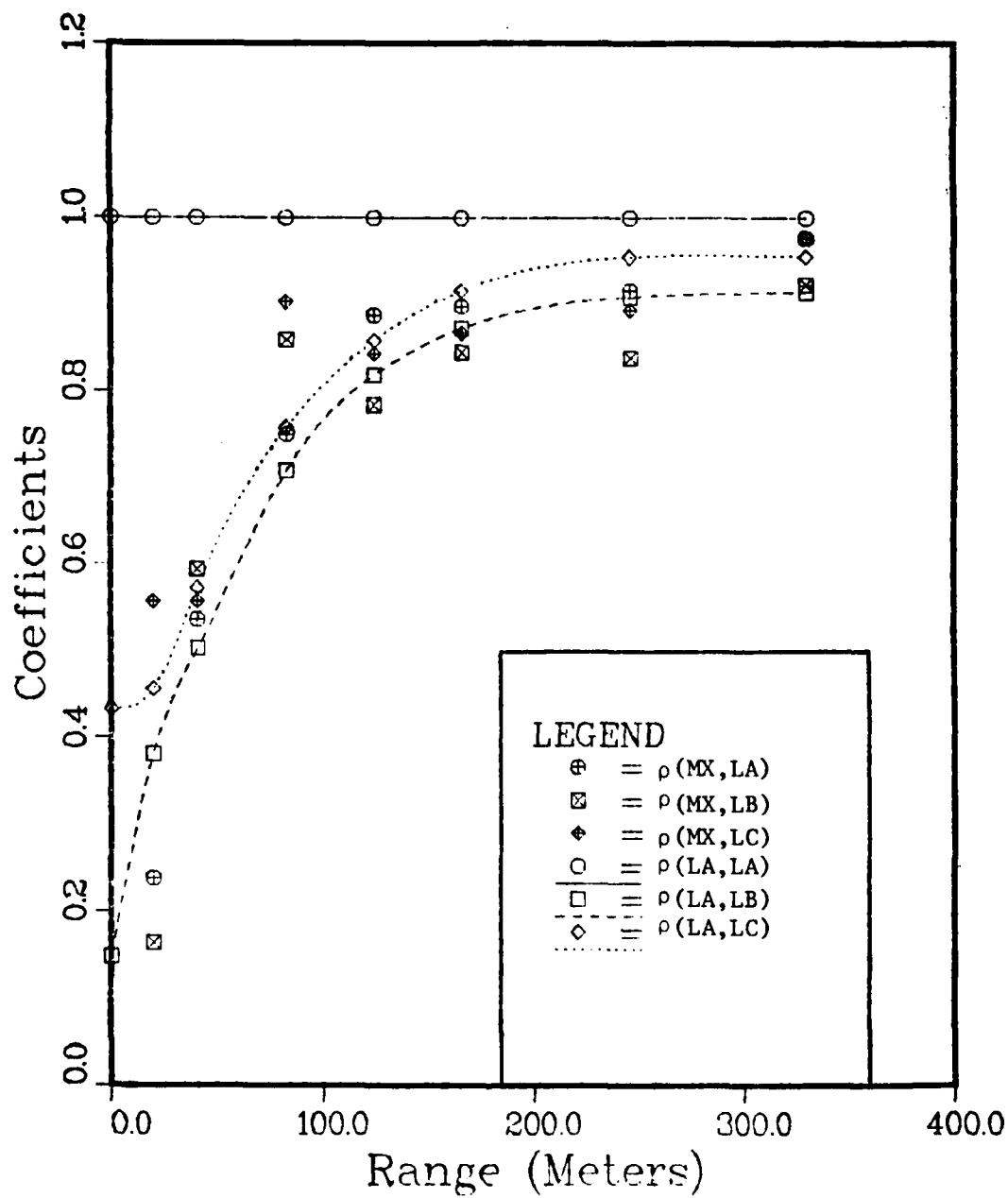


Figure 30. Measured matrix, MX100 (unknown: Source X) vs library coefficients of row 1.

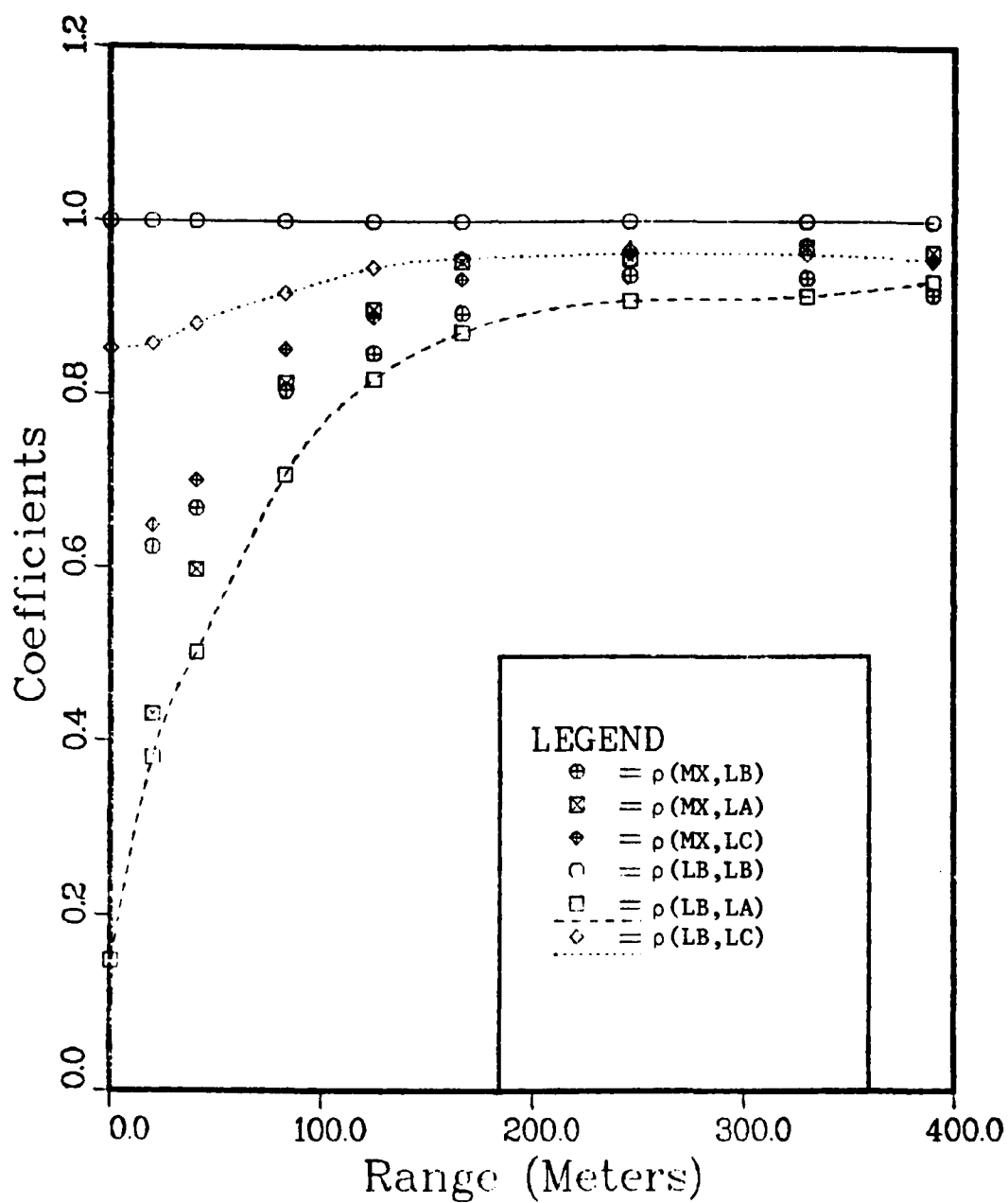


Figure 31. Measured matrix, MX (unknown: Source X) vs library coefficients of row 2.

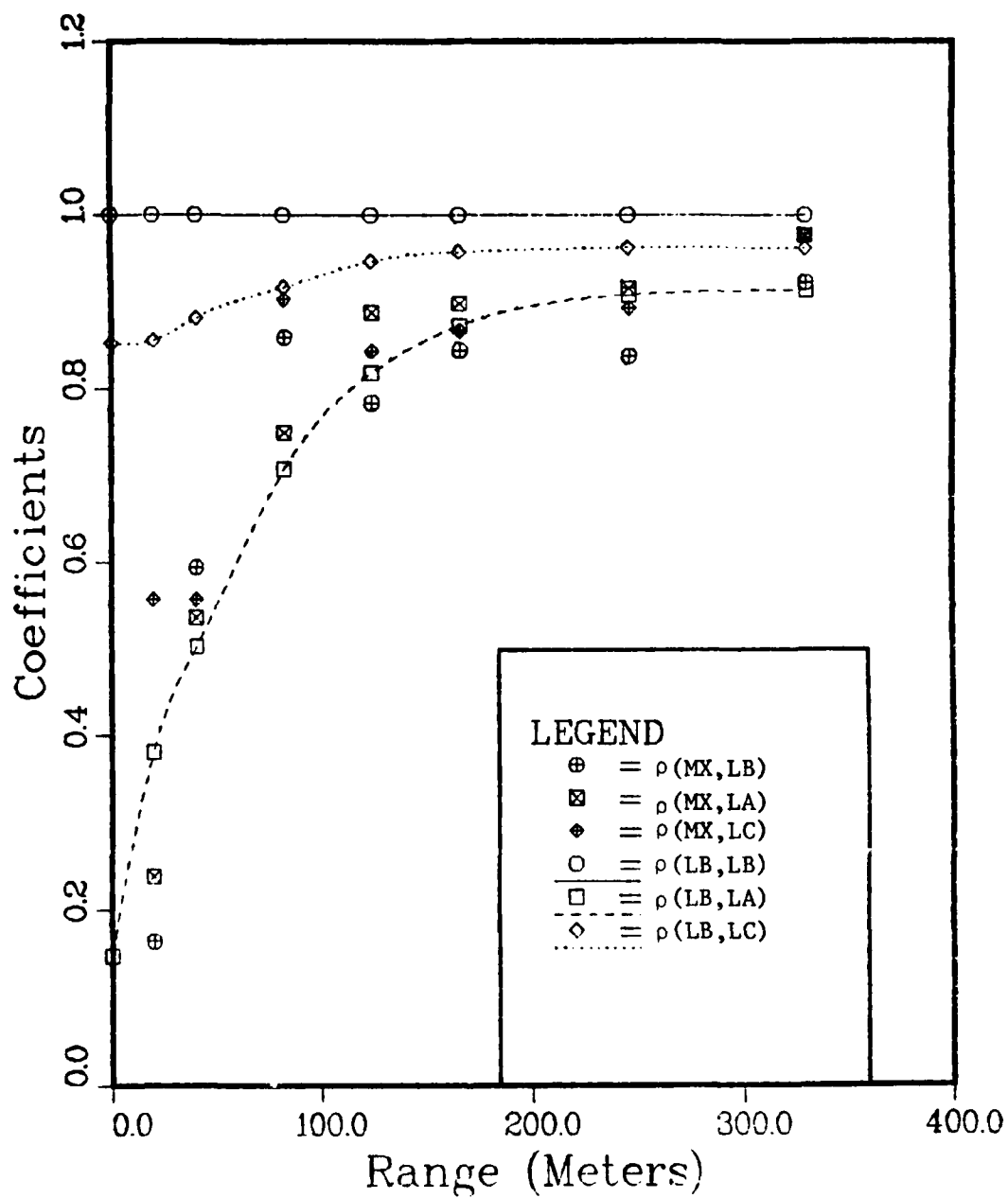


Figure 32. Measured matrix, MX100 (unknown: Source X) vs library coefficients of row 2.

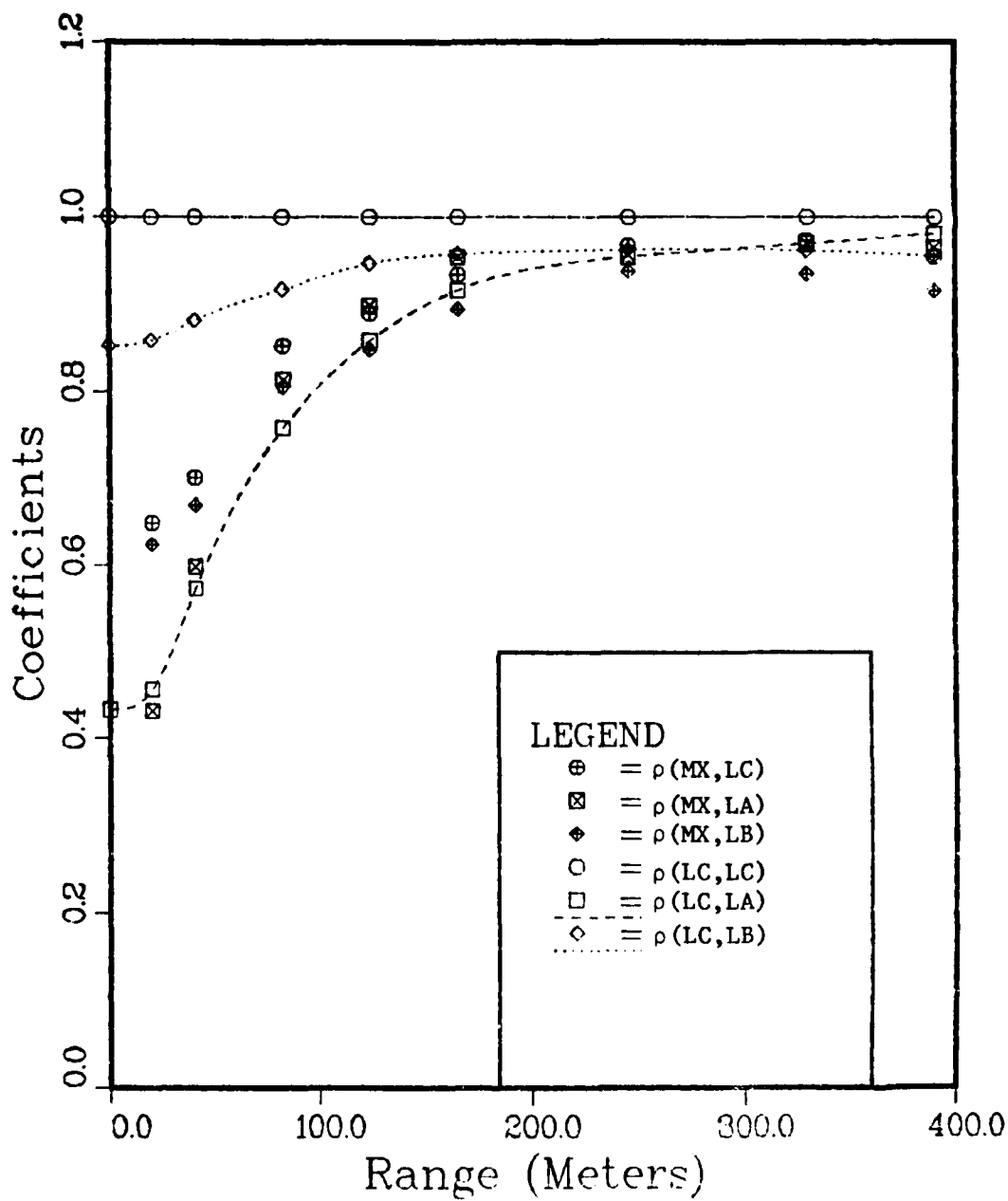


Figure 33. Measured matrix, MX (unknown: Source X) vs library coefficients of row 3.

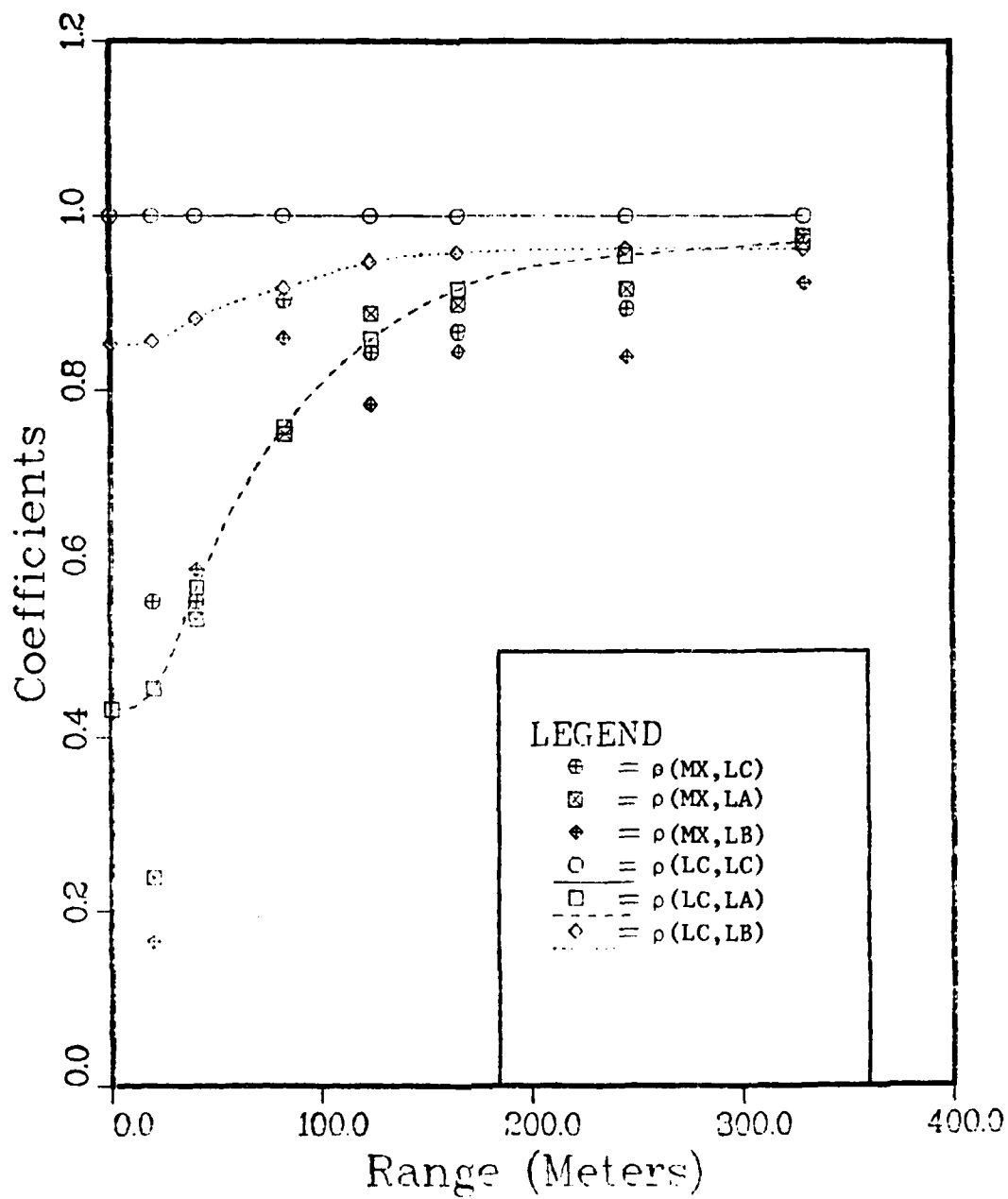


Figure 34. Measured matrix, MX100 (unknown: Source X) vs library coefficients of row 3.

What is needed is some method able to discriminate Source X as a "new" source.

Examining the coefficients of Table X shows that there is no clear "dominant" coefficient at the low range, i.e. less than 124.5 meters. The magnitudes at these low ranges differ from the large values found from the "good" and "poor" coefficients for the correct source. To argue that these low values indicate a "new" source would be hard to justify. Even if this statement could be justified, this low range is within the photopeak range and therefore, adds little to the analysis of X spectra.

#### Detection Time

The minimum count time based on a spectrum consisting of a total of 1000 source photons will be determined. The count time determined is the minimum since the same results are desired as the ones with the "good" spectra. The 1000 photons were chosen due to the simple fact that 1000 source photons were used as input for the generation of the measured spectra.

The minimum count time will be based on a g-sec basis so that knowledge of the source's mass can quickly give the minimum count time needed.

This problem will be based on the source spectrum of B. The source spectra of B was originally based on a 5:1 mass ratio of U:Pu. From the relative amounts of the source spectrum, a source intensity may be found. Table XI shows the finding of the total intensity of Source B.

Table XI. Total intensity determination  
of source B

<u>ENERGY BIN (KeV)</u>	<u>SOURCE SPECTRUM (%)</u>	<u>MATERIAL</u>	<u>INTENSITY (1/g. sec.)</u>	<u>EFFECTIVE INTENSITY (1/g. sec.)</u>
450	.1043	Pu <sup>239</sup>	3.4E4	3.55E3
400				3.4E3
300	.100	Pu <sup>239</sup>	3.4E4	3.4E3
200	.796	U <sup>235</sup>	4.3E4	2.85E3
100		Pu : $\frac{6}{1}$		1.4E5
TOTAL:				5.4E4/g. sec.

The minimum count time can be determined by the following  
equation,

$$\text{Count Time} = \frac{1000 * 4\pi(\text{Range})^2}{I_o * \text{Area}} \quad (1)$$

where

$I_o$  = Total intensity of source in 1/g. sec.

Area = Area of detector face

For source B  $I_o = 5.4E4$  photons/g. sec.

substituting into (1)

$$\text{Count Time} = \frac{.232(\text{Range})^2}{\text{Area}} \quad \text{g. sec.}$$

Figure 35 plots the count time vs. range for a detector area of 5 cm<sup>2</sup>.



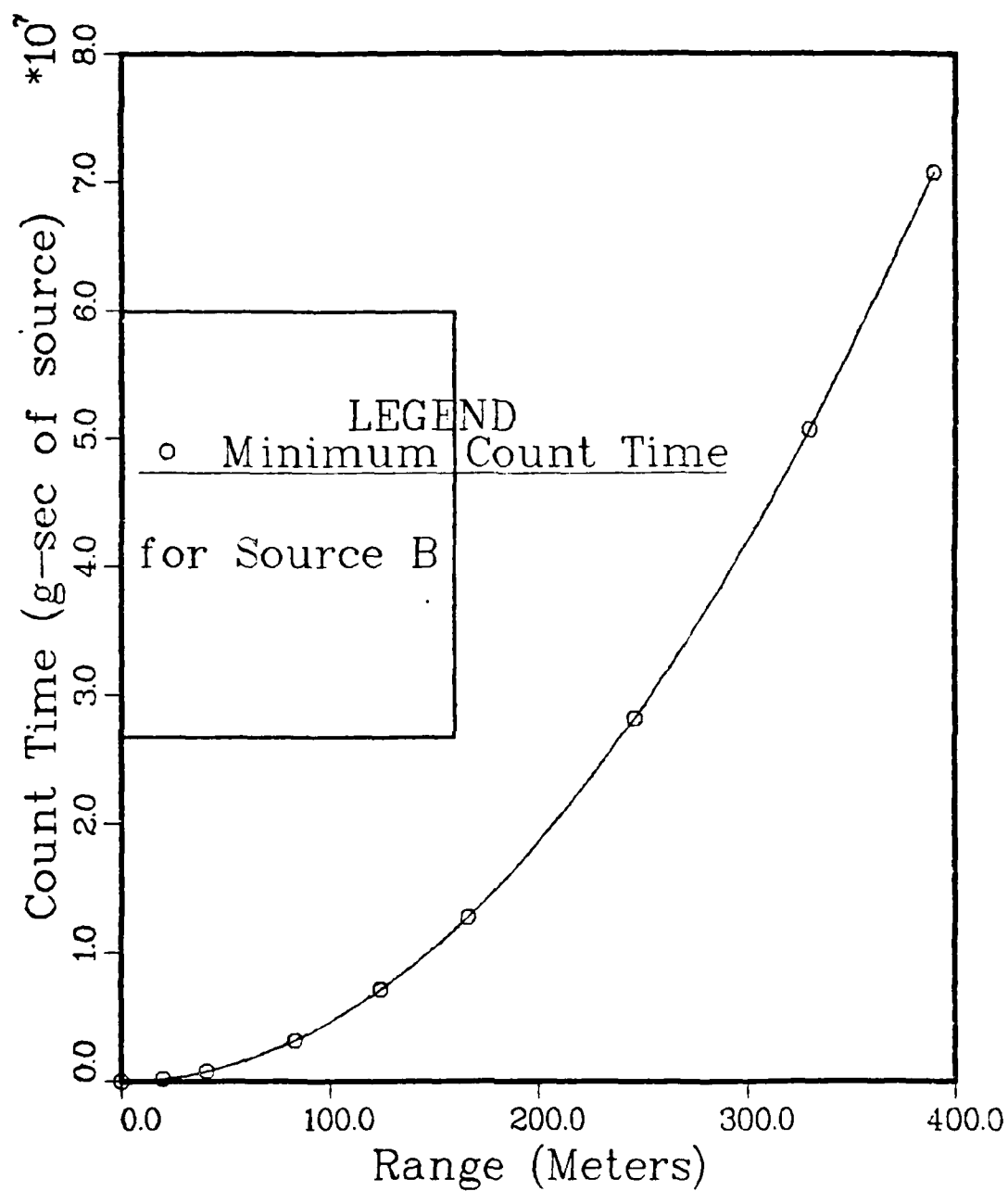


Figure 35. Detection Plot

## V. CONCLUSIONS AND RECOMMENDATIONS

### Conclusions

The concept of utilizing entire spectral features for source identification has been investigated. The study used a simple correlation function and employed two decision rules for determining the source of the unknown spectra. Two scenarios were investigated: (1) each measured spectrum has a source in the library, and (2) the measured spectrum's source is not a library source.

When the decisions rules are applied to scenario (1) the following conclusions are drawn:

(1). The counting of collided photons as well as uncollided photons seems to add structure to the spectrum which is unique to the source. The added structure extends identification further than the photopeak identification.

(2). The proposed techniques for identifying unknown sources have merit as a possible alternative to photopeak identification based on their success for choosing the correct sources.

(3). The "matching" of a statistically "poor" spectrum still gives results that are correct in most cases studied.

When the decisions rules are applied to scenario (2) the following conclusions are drawn:

(1). The small values of the measured coefficients "might" indicate that the measured source is not a library source. But these low values are indicated for ranges within the photopeak range.

(2). Based on the decision rules the non-library spectra cannot be

discriminated as a "new" source.

Lastly, a minimum count time was determined for source B as a function of range.

### Recommendations

Based on the assumptions and observations while working on this thesis the following recommendations are proposed:

(1). The modeling of a more realistic detector with appropriate response functions should be considered. However, the use of Morse for photon transport is not suggested because of its complexity. The only reason Morse should be used is to model complex 3-D geometries which can not be done by more conventional transport codes.

(2). One of the underlying assumptions throughout this study has been that a detector which allows both the collided and uncollided photons produces a spectrum which can be used for extending identification. As mentioned most detection schemes want to reduce background (scattered photons included) and only get peak structure.

A study of possible detector types and designs which could be used should be investigated.

(3). Using the simple correlation equation on the group structure of the spectra was a manageable task. Applying the proposed techniques to real data would involve a complicated process. This process would consist of setting up a data acquisition system (a personal computer might suffice but it would be limited by storage capabilities) and the proper analysis routine (computer program to solve for the coefficients). The analysis system would involve finding the proper correlation technique to handle the large amount of data due to the large number of

channels in todays multichannel analyzers, i.e. finding coefficients on a group to group basis would be extremely costly in computer resources. This work would lay the foundation for further nuclear spectral analysis studies.

(5). The entire analysis used a 3 Source library. Does the identification range increase as the library becomes larger? Do the decision rules still compare with one another in their success rate? These questions are the basis for further study for determining correlation limitations for the proposed decision rules.

## Appendices

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Appendix A. Photon Energy Group Boundaries  
for DLC-48/PVC. (Ref )

<u>Group Number</u>	<u>Energy (MeV)</u>
1	14.0
2	12.0
3	10.0
4	8.0
5	7.5
6	7.0
7	6.5
8	6.0
9	5.5
10	5.0
11	4.5
12	4.0
13	3.5
14	3.0
15	2.5
16	2.0
17	1.66
18	1.50
19	1.33
20	1.0
21	0.80
22	0.70
23	0.60
24	0.512
25	0.510
26	0.45
27	0.40
28	0.30
29	0.20
30	0.15
31	0.10
32	0.075
33	0.060
34	0.045
35	0.030
36	0.020
	0.010

## Appendix B

### A. Variance of Library Coefficients, $\sigma_{\bar{\rho}_{AB}}^2 = \sigma_{AB}^2$

The variance,  $\sigma_{A,B}^2$ , is determined by the following:

$$\text{Recall } \rho_{AB} = \sum_{i=1}^N C_i^A C_i^B$$

$$\text{But actual } C_i^A = \bar{C}_i^A + \zeta_i^A$$

where

$C_i$  = estimated means

$$C_i^B = \bar{C}_i^B + \zeta_i^B$$

$\bar{C}_i$  = true means

$\zeta_i$  = random error

The expectation of  $\rho_{AB}$  is

$$\begin{aligned} E \{ \rho_{AB} \} &= E \left\{ \sum_i (\bar{C}_i^A + \zeta_i^A) (\bar{C}_i^B + \zeta_i^B) \right\} \\ \bar{\rho}_{AB} &= E \left\{ \sum_i \left( \bar{C}_i^A \bar{C}_i^B + \bar{C}_i^B \zeta_i^A + \bar{C}_i^A \zeta_i^B + \zeta_i^A \zeta_i^B \right) \right\} \end{aligned} \quad (1)$$

now,

$$E \left\{ \bar{C}_i^A \zeta_i^B \right\} \text{ and } E \left\{ \bar{C}_i^B \zeta_i^A \right\} = 0$$

Because it is equally likely that the different  $\zeta_i$ s will be positive as well as negative, therefore the average (or expectation) will be zero.

(1) Therefore reduces to

$$\bar{\rho}_{A,B} = \sum_i \left( \bar{C}_i^A \bar{C}_i^B + \delta^{AB} E \left\{ \zeta_i^A \zeta_i^B \right\} \right) \quad (2)$$

if  $A=B$  then  $\delta^{AB}=1$

if  $A \neq B$  then  $\delta^{AB}=0$

If  $A=B$  then

$$E \left\{ \zeta_1^A \zeta_1^B \right\} = \sigma_1^{A^2} \text{ or } \sigma_1^{B^2}$$

$\sigma_{AB}^2$  is defined as

$$\begin{aligned} \sigma_{AB}^2 &= E \left\{ \left( \rho_{AB} - \bar{\rho}_{AB} \right)^2 \right\} \\ &= E \left\{ \left( \rho_{AB}^2 - 2\bar{\rho}_{AB}\rho_{AB} + \bar{\rho}_{AB}^2 \right) \right\} \end{aligned}$$

the  $E$  operator is linear so

$$\sigma_{AB}^2 = E \left\{ \rho_{AB}^2 \right\} - 2\bar{\rho}_{AB} E \left\{ \rho_{AB} \right\} + \bar{\rho}_{AB}^2$$

therefore

$$\sigma_{AB}^2 = E \left\{ \rho_{AB}^2 \right\} - \bar{\rho}_{AB}^2 \quad (3)$$

Taking the terms individually.

$\bar{\rho}_{AB}^2$ :

$$\left( \bar{\rho}_{AB} \right)^2 = \left( \sum_i \bar{C}_i^A \bar{C}_i^B + \sum_i \delta_i^{AB} \zeta_i^A \zeta_i^B \right)^2$$

Case  $A \neq B$

$$\left( \bar{\rho}_{AB} \right)^2 = \sum_{i,j} \bar{C}_i^A \bar{C}_i^B \bar{C}_j^A \bar{C}_j^B$$

Case  $A=B$

$$\left( \bar{\rho}_{AB} \right)^2 = \sum_i \left( \bar{C}_i^A \bar{C}_i^A + \sigma_i^{A^2} \right) \sum_j \left( \bar{C}_j^A \bar{C}_j^A + \sigma_j^{A^2} \right)$$



$$= \sum_{i,j} \bar{C}_i^A \bar{C}_i^A \bar{C}_j^A \bar{C}_j^A + 2 \sum_{i,j} \bar{C}_i^A \bar{C}_i^A \sigma_j^{A^2} + \sum_{i,j} \sigma_i^{A^2} \sigma_j^{A^2}$$

$$\frac{E \rho_{AB}^2}{E \left\{ \rho_{AB}^2 \right\}} :$$

$$E \left\{ \rho_{AB}^2 \right\} = E \left\{ \sum_i (\bar{C}_i^A + \zeta_i^A) (\bar{C}_i^B + \zeta_i^B) \sum_j (\bar{C}_j^A + \zeta_j^A) (\bar{C}_j^B + \zeta_j^B) \right\}$$

Case A≠B

$$\begin{aligned} E \left\{ \rho_{AB}^2 \right\} &= \sum_{i,j} \bar{C}_i^A \bar{C}_i^B \bar{C}_j^A \bar{C}_j^B + \sum_i (\bar{C}_i^A)^2 \sigma_i^{B^2} \\ &\quad + \sum_i (\bar{C}_i^B)^2 \sigma_i^{A^2} + \sum_i \sigma_i^{A^2} \sigma_i^{B^2} \end{aligned}$$

Case A=B

$$\begin{aligned} E \left\{ \rho_{AB}^2 \right\} &= \sum_{i,j} \bar{C}_i^A \bar{C}_i^A \bar{C}_j^A \bar{C}_j^A + 2 \sum_{i,j} \bar{C}_i^A \bar{C}_i^A \sigma_j^{A^2} \\ &\quad + 4 \sum_i \bar{C}_i^A \bar{C}_i^A \sigma_i^{A^2} + \sum_{i,j} \sigma_i^{A^2} \sigma_j^{A^2} \end{aligned}$$

Therefore

Case A≠B

$$\sigma_{AB}^2 = \sum_i (\bar{C}_i^A)^2 \sigma_i^{B^2} + \sum_i (\bar{C}_i^B)^2 \sigma_i^{A^2} + \sum_i (\sigma_i^{A^2}) (\sigma_i^{B^2})$$

Case A=B

$$\sigma_{AA}^2 = 4 \sum_i \bar{C}_i^{A^2} \sigma_i^{A^2}$$

The standard deviation is  $\sqrt{\sigma^2}$ . The standard deviations are actually standard deviations of the means since the  $\sigma_i$ 's were of the mean. This

is just for the library.

B. Variance of Library Coefficients,  $\sigma_{\rho_{M,L}}^2$ .

There are two types of error to consider:

(1) The standard error of the mean associated with estimating the fluence values, i.e.  $C_i^L = \bar{C}_i + \zeta_i^L$  where  $\zeta_i^L$  is the random error of the mean.

(2) The error associated with the measured fluence  $C_i^M, \zeta_i^M$  which indicates the deviation of  $C_i^M$  about  $\bar{C}_i^L$ .

To find  $\sigma_{M,L}$

$$\begin{aligned}\bar{\rho}_{ML} &= E \rho_{ML} = E \left\{ \sum_i (\bar{C}_i^L + \zeta_i^L) (\bar{C}_i^M + \zeta_i^M) \right\} \\ &= \sum_i \bar{C}_i^L \bar{C}_i^M \\ \left\{ \bar{\rho}_{ML} \right\}^2 &= \sum_{i,j} \bar{C}_i^L \bar{C}_i^M \bar{C}_j^L \bar{C}_j^M \\ E \left\{ \rho_{ML}^2 \right\} &= E \left\{ \sum_i (\bar{C}_i^L + \zeta_i^L) (\bar{C}_i^M + \zeta_i^M) \sum_j (\bar{C}_j^L + \zeta_j^L) (\bar{C}_j^M + \zeta_j^M) \right\}\end{aligned}$$

This reduces to

$$\begin{aligned}E \left\{ \rho_{ML}^2 \right\} &= \sum_{i,j} \bar{C}_i^L \bar{C}_i^M \bar{C}_j^L \bar{C}_j^M + \sum_i (\bar{C}_i^L)^2 \sigma_i^{M^2} + \sum_i (\bar{C}_i^M)^2 \sigma_i^{L^2} \\ &\quad + \sum_i \sigma_i^{L^2} \sigma_i^{M^2}\end{aligned}$$

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STUDY OF TOTAL GAMMA SPECTRA CORRELATION FOR EXTENDING

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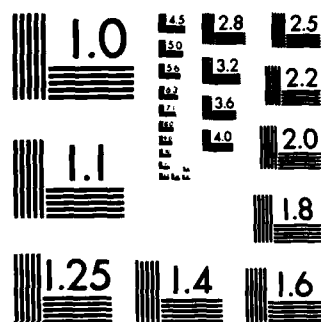
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Therefore

$$\begin{aligned}\sigma_{ML}^2 &= E \left\{ \left( \rho_{ML} - \bar{\rho}_{ML} \right)^2 \right\} \\ &= \sum_i \left( \bar{C}_i^L \right)^2 \sigma_i^{M^2} + \sum_i \left( \bar{C}_i^M \right)^2 \sigma_i^{L^2} + \sum_i \sigma_i^{L^2} \sigma_i^{M^2}\end{aligned}$$

Now  $\bar{C}_i^M$  must be estimated by assuming a source L such that

$\bar{C}_i^L = \bar{C}_i^M$  since no estimate of  $\bar{C}_i^L$  is available, i.e.

$$C_i^M \left( \begin{array}{l} \text{from single} \\ \text{batch} \end{array} \right) \neq \bar{C}_i^L$$

while

$$C_i^L \left( \begin{array}{l} \text{avg estimate after} \\ \text{many batches} \end{array} \right) \approx \bar{C}_i^L$$

In a similar fashion  $\sigma_i^{M^2}$  must be estimated. Since  $\sigma_i^{M^2}$  corresponds to the variance of the data and not the mean, the relationship between the assumed library source is

$$\sigma_i^{L^2} = \frac{\sigma_i^{M^2}}{N}$$

where N = number of batches used in estimating the fluence mean.

For the 1000 photon case

$$\sigma_i^{M^2} = N \sigma_i^{L^2}$$

and for the 100 case

$$\sigma_i^{M^2} = 10 * N * \sigma_i^{L^2}$$

Since counting statistics says that the estimates should vary as  
the  $\sqrt{\text{intensity}}$ .

Appendix C. Distribution of measured estimates about their library estimates.

LEGEND FOR APPENDIX C

Bin (KeV)	Library (Counts/Photon) Standard Deviation of the Mean (s.d.m.)	Measured N/A	Number of s.d. values are from library value
Bin (KeV)			

GROUP	COUNTS/PHOTON LIBRARY A	RANGE=20. M MEASURED A-1000	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	5.940E-002 7.477E-003	6.195E-002 0.000E+000	-0.340E+000
800	3.043E-002 6.243E-003	3.540E-002 0.000E+000	-0.809E+000
700	2.947E-001 1.771E-002	2.894E-001 0.000E+000	3.008E-001
600	1.161E-002 5.980E-003	9.587E-003 0.000E+000	3.387E-001
512	1.011E-001 0.100E-002	1.000E-001 0.000E+000	3.375E-003
510	1.066E-002 6.232E-003	1.551E-002 0.000E+000	5.059E-001
450	2.472E-002 7.072E-003	1.635E-002 0.000E+000	1.064E+000
400	7.068E-002 1.371E-002	6.534E-002 0.000E+000	3.890E-001
300	6.154E-002 1.512E-002	5.293E-002 0.000E+000	5.696E-001
200	1.183E-001 1.764E-002	1.404E-001 0.000E+000	-0.125E+001
150	9.520E-002 1.637E-002	7.627E-002 0.000E+000	1.162E+000
100	1.492E-002 0.727E-003	3.563E-002 0.000E+000	-0.237E+001
75	2.003E-002 5.462E-003	1.439E-002 0.000E+000	1.180E+000
60	2.531E-002 7.334E-003	1.687E-002 0.000E+000	1.151E+000
45	1.524E-002 6.370E-003	6.451E-003 0.000E+000	1.378E+000
30	3.897E-004 9.040E-004	0.000E+000 0.000E+000	4.306E-001
20	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000



GROUP	COUNTS/PHOTON LIBRARY A	RANGE=41 M MEASURED A-1000	DEVIATION FROM MEAN
1330	0.000E+000	0.000E+000	0.000E+000
	0.000E+000	0.000E+000	
1000	5.024E-002	4.261E-002	9.671E-001
	7.093E-003	0.000E+000	
800	2.684E-002	3.133E-002	-7.90E+000
	5.682E-003	0.000E+000	
700	2.449E-001	2.437E-001	0.068E-002
	1.328E-002	0.000E+000	
600	2.253E-002	2.676E-002	-5.31E+000
	7.966E-003	0.000E+000	
512	1.437E-001	1.555E-001	-1.16E+001
	1.014E-002	0.000E+000	
510	3.117E-002	2.265E-002	1.056E+000
	0.071E-003	0.000E+000	
450	3.306E-002	4.059E-002	-9.42E+000
	7.907E-003	0.000E+000	
400	0.656E-002	9.557E-002	-5.91E+000
	1.522E-002	0.000E+000	
300	9.438E-002	9.483E-002	-2.79E-001
	1.622E-002	0.000E+000	
200	1.205E-001	1.041E-001	9.210E-001
	1.701E-002	0.000E+000	
150	1.200E-001	1.252E-001	-2.41E+000
	2.171E-002	0.000E+000	
100	4.043E-002	4.500E-002	-3.36E+000
	1.599E-002	0.000E+000	
75	3.013E-002	9.704E-003	1.058E+000
	1.095E-002	0.000E+000	
60	2.934E-002	1.942E-002	9.317E-001
	1.065E-002	0.000E+000	
45	1.500E-002	5.400E-003	1.698E+000
	6.077E-003	0.000E+000	
30	7.779E-004	1.571E-003	-9.66E+000
	0.206E-004	0.000E+000	
20	1.600E-005	0.000E+000	2.393E-001
	6.720E-005	0.000E+000	
10	0.000E+000	0.000E+000	0.000E+000
	0.000E+000	0.000E+000	

GROUP	COUNTS/PHOTON LIBRARY A	RANGE=83 M MEASURED A-1000	DEVIATION FROM MEAN
1330	0.000E+000	0.000E+000	0.000E+000
1000	0.000E+000	0.000E+000	0.000E+000
	3.950E-002	3.715E-002	
800	5.456E-003	0.000E+000	-.288E+000
	2.122E-002	2.581E-002	
700	4.814E-003	0.000E+000	-.954E+000
	1.677E-001	1.814E-001	
600	1.117E-002	0.000E+000	-.122E+001
	3.517E-002	4.046E-002	
512	1.810E-002	0.000E+000	-.522E+000
	8.760E-002	9.265E-002	
510	7.790E-003	0.000E+000	-.647E+000
	3.923E-002	3.854E-002	
450	9.390E-003	0.000E+000	7.317E-002
	4.000E-002	3.826E-002	
400	1.039E-002	0.000E+000	1.672E-001
	1.021E-001	9.841E-002	
300	1.937E-002	0.000E+000	1.910E-001
	1.260E-001	1.172E-001	
200	2.110E-002	0.000E+000	4.166E-001
	1.198E-001	9.147E-002	
150	2.000E-002	0.000E+000	1.415E+000
	1.644E-001	1.381E-001	
100	2.397E-002	0.000E+000	1.895E+000
	7.309E-002	8.836E-002	
75	2.097E-002	0.000E+000	-.728E+000
	4.653E-002	4.727E-002	
60	1.423E-002	0.000E+000	-.516E-001
	4.580E-002	6.162E-002	
45	1.523E-002	0.000E+000	-.103E+001
	2.174E-002	2.269E-002	
30	1.164E-002	0.000E+000	-.819E-001
	1.177E-003	5.487E-003	
20	1.303E-003	0.000E+000	-.338E+001
	0.000E+000	0.000E+000	
10	0.000E+000	0.000E+000	0.000E+000
	0.000E+000	0.000E+000	
	0.000E+000	0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY A	RANGE=124.5 M MEASURED A-1000	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	2.633E-002 4.922E-003	2.302E-002 0.000E+000	6.723E-001
800	1.573E-002 3.622E-003	1.288E-002 0.000E+000	7.875E-001
700	1.181E-001 1.023E-002	1.434E-001 0.000E+000	-.246E+001
600	3.285E-002 6.676E-003	2.869E-002 0.000E+000	6.230E-001
512	5.572E-002 4.431E-003	5.350E-002 0.000E+000	5.020E-001
510	3.796E-002 8.092E-003	4.314E-002 0.000E+000	-.640E+000
450	3.674E-002 7.750E-003	4.949E-002 0.000E+000	-.164E+001
400	9.565E-002 1.526E-002	8.142E-002 0.000E+000	9.330E-001
300	1.350E-001 2.222E-002	1.422E-001 0.000E+000	-.326E+000
200	1.163E-001 2.000E-002	1.120E-001 0.000E+000	2.179E-001
150	1.704E-001 3.312E-002	1.560E-001 0.000E+000	4.347E-001
100	1.002E-001 3.019E-002	6.807E-002 0.000E+000	6.656E-001
75	7.551E-002 2.045E-002	3.912E-002 0.000E+000	1.779E+000
60	6.651E-002 2.312E-002	7.492E-002 0.000E+000	-.363E+000
45	3.209E-002 1.357E-002	2.958E-002 0.000E+000	1.850E-001
30	2.036E-003 2.172E-003	2.207E-003 0.000E+000	-.115E+000
20	3.014E-006 1.489E-005	0.000E+000 0.000E+000	2.562E-001
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY A	RANGE=166 M MEASURED A-1000	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	1.914E-002 4.429E-003	1.747E-002 0.000E+000	3.762E-001
800	1.127E-002 3.860E-003	1.541E-002 0.000E+000	-1.107E+001
700	8.123E-002 6.779E-003	8.154E-002 0.000E+000	-1.465E-001
600	2.880E-002 5.770E-003	1.637E-002 0.000E+000	2.153E+000
512	3.555E-002 3.301E-003	3.283E-002 0.000E+000	8.251E-001
510	3.514E-002 5.487E-003	3.740E-002 0.000E+000	-1.411E+000
450	3.432E-002 7.147E-003	3.121E-002 0.000E+000	4.352E-001
400	8.816E-002 1.501E-002	9.856E-002 0.000E+000	-1.693E+000
300	1.234E-001 2.135E-002	1.199E-001 0.000E+000	1.600E-001
200	9.955E-002 2.160E-002	1.335E-001 0.000E+000	-1.157E+001
150	1.635E-001 2.622E-002	1.444E-001 0.000E+000	7.269E-001
100	1.172E-001 3.273E-002	9.454E-002 0.000E+000	6.932E-001
75	7.679E-002 2.010E-002	5.357E-002 0.000E+000	1.155E+000
60	8.765E-002 2.939E-002	1.196E-001 0.000E+000	-1.108E+001
45	4.260E-002 1.933E-002	3.550E-002 0.000E+000	3.676E-001
30	2.912E-003 2.051E-003	4.286E-003 0.000E+000	-1.669E+000
20	1.739E-006 5.440E-006	0.000E+000 0.000E+000	3.197E-001
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

COUNTS/PHOTON GROUP	LIBRARY A	RANGE=245 M MEASURED A-1000	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	1.074E-002 2.152E-003	1.161E-002 0.000E+000	-.402E+000
800	6.137E-003 2.024E-003	1.101E-002 0.000E+000	-.240E+001
700	4.007E-002 5.292E-003	3.114E-002 0.000E+000	1.687E+000
600	2.041E-002 4.566E-003	2.263E-002 0.000E+000	-.485E+000
512	1.438E-002 1.957E-003	1.353E-002 0.000E+000	4.296E-001
510	2.352E-002 4.076E-003	2.086E-002 0.000E+000	5.462E-001
450	2.371E-002 4.599E-003	2.138E-002 0.000E+000	5.072E-001
400	6.009E-002 1.226E-002	5.043E-002 0.000E+000	2.006E-001
300	9.273E-002 1.944E-002	7.965E-002 0.000E+000	6.729E-001
200	6.402E-002 1.014E-002	5.602E-002 0.000E+000	0.602E-001
150	1.292E-001 2.203E-002	1.380E-001 0.000E+000	-.417E+000
100	9.004E-002 1.954E-002	0.652E-002 0.000E+000	2.213E-001
75	7.496E-002 4.116E-002	4.939E-002 0.000E+000	6.212E-001
60	0.839E-002 3.049E-002	6.220E-002 0.000E+000	0.591E-001
45	4.720E-002 1.471E-002	2.833E-002 0.000E+000	1.203E+000
30	4.174E-003 5.119E-003	5.672E-004 0.000E+000	7.046E-001
20	1.234E-006 4.257E-006	0.000E+000 0.000E+000	2.090E-001
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY A	RANGE=333 M MEASURED A-1000	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	5.400E-003 1.107E-003	4.063E-003 0.000E+000	5.263E-001
800	3.503E-003 1.100E-003	4.720E-003 0.000E+000	-.103E+001
700	1.985E-002 2.275E-003	2.198E-002 0.000E+000	-.930E+000
600	1.132E-002 2.533E-003	1.494E-002 0.000E+000	-.143E+001
512	5.400E-003 1.365E-003	4.131E-003 0.000E+000	9.004E-001
510	1.405E-002 3.274E-003	1.374E-002 0.000E+000	3.394E-001
450	1.346E-002 3.707E-003	2.036E-002 0.000E+000	-.106E+001
400	3.959E-002 5.297E-003	5.570E-002 0.000E+000	-.304E+001
300	6.300E-002 3.110E-002	5.100E-002 0.000E+000	4.110E-001
200	4.240E-002 9.091E-003	3.263E-002 0.000E+000	9.962E-001
150	0.336E-002 1.699E-002	9.076E-002 0.000E+000	-.435E+000
100	6.455E-002 1.353E-002	5.607E-002 0.000E+000	5.676E-001
75	6.147E-002 2.301E-002	9.700E-002 0.000E+000	-.154E+001
60	6.400E-002 2.057E-002	1.219E-001 0.000E+000	-.200E+001
45	3.106E-002 1.192E-002	4.600E-002 0.000E+000	-.110E+001
30	2.376E-003 2.467E-003	1.006E-003 0.000E+000	5.554E-001
20	5.473E-006 1.811E-005	0.000E+000 0.000E+000	3.023E-001
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

COUNTS/PHOTON GROUP	LIBRARY A	RANGE=390 M MEASURED A-1000	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	3.045E-003 0.000E+000	0.000E+000
1000	3.407E-003 1.250E-003	7.074E-003 0.000E+000	-.291E+001
800	2.509E-003 1.073E-003	3.243E-003 0.000E+000	-.870E+000
700	1.172E-002 2.025E-003	5.464E-003 0.000E+000	2.215E+000
600	0.101E-003 2.723E-003	7.410E-003 0.000E+000	2.001E-001
512	2.726E-003 6.445E-004	0.000E+000 0.000E+000	4.230E+000
510	9.002E-003 2.233E-003	4.642E-003 0.000E+000	2.347E+000
450	1.035E-002 3.952E-003	3.304E-003 0.000E+000	1.762E+000
400	2.504E-002 6.096E-003	2.301E-002 0.000E+000	2.952E-001
300	3.061E-002 9.940E-003	3.632E-002 0.000E+000	2.307E-001
200	3.106E-002 1.043E-002	3.314E-002 0.000E+000	-.199E+000
150	6.159E-002 2.140E-002	4.619E-002 0.000E+000	7.170E-001
100	4.941E-002 2.300E-002	6.110E-002 0.000E+000	-.553E+000
75	4.007E-002 1.103E-002	5.592E-002 0.000E+000	-.134E+001
60	5.202E-002 1.602E-002	3.813E-002 0.000E+000	0.675E-001
45	2.239E-002 5.772E-003	2.369E-002 0.000E+000	-.225E+000
30	2.440E-003 2.224E-003	0.904E-004 0.000E+000	6.930E-001
20	0.009E-006 2.466E-005	0.000E+000 0.000E+000	3.401E-001
10	0.000E+000 2.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY B	RANGE=20. M MEASURED B-1000	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
800	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
700	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
600	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
512	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
510	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
450	7.940E-002 1.003E-002	9.359E-002 0.000E+000	-.141E+001
400	9.463E-002 1.057E-002	1.026E-001 0.000E+000	-.754E+000
300	2.445E-002 1.124E-002	1.463E-002 0.000E+000	0.735E-001
200	6.606E-001 1.677E-002	6.577E-001 0.000E+000	1.757E-001
150	1.759E-001 4.177E-002	1.504E-001 0.000E+000	4.191E-001
100	3.174E-002 1.502E-002	3.165E-002 0.000E+000	-.760E-002
75	4.079E-003 6.700E-003	1.351E-002 0.000E+000	-.127E+001
40	1.660E-003 4.435E-003	1.270E-002 0.000E+000	-.250E+001
45	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
30	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
20	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000



GROUP	COUNTS/PHOTON LIBRARY C	RANGE=20. M MEASURED C-1000	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	2.205E-001 1.450E-002	2.261E-001 0.000E+000	- .388E+000
800	1.121E-001 9.743E-003	1.194E-001 0.000E+000	- .752E+000
700	3.011E-002 5.562E-003	3.605E-002 0.000E+000	- .186E+001
600	0.795E-003 4.728E-003	1.630E-002 0.000E+000	- .158E+001
512	4.417E-004 1.042E-003	2.129E-003 0.000E+000	- .162E+001
510	6.038E-003 4.270E-003	8.227E-003 0.000E+000	- .325E+000
450	5.911E-003 3.707E-003	1.374E-003 0.000E+000	1.198E+000
400	1.296E-002 0.071E-003	1.941E-002 0.000E+000	- .799E+000
300	2.110E-002 0.903E-003	2.651E-002 0.000E+000	- .602E+000
200	4.168E-001 1.017E-002	4.354E-001 0.000E+000	- .102E+001
150	1.536E-001 2.171E-002	1.060E-001 0.000E+000	- .149E+001
100	4.609E-002 1.228E-002	4.018E-002 0.000E+000	5.460E-001
75	0.653E-003 7.200E-003	9.836E-003 0.000E+000	- .171E+000
60	2.327E-003 5.245E-003	0.000E+000 0.000E+000	4.436E-001
45	3.638E-004 1.625E-003	0.000E+000 0.000E+000	2.239E-001
30	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
20	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY B	RANGE=03 M MEASURED B-1000	DEVIATION FROM MEAN
1338	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
800	1.651E-005 9.044E-005	0.000E+000 0.000E+000	1.026E-001
700	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
600	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
512	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
510	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
450	4.199E-002 6.070E-003	4.150E-002 0.000E+000	6.745E-002
400	6.441E-002 7.074E-003	5.040E-002 0.000E+000	1.779E+000
300	4.505E-002 1.179E-002	4.433E-002 0.000E+000	6.034E-002
200	3.559E-001 1.779E-002	3.467E-001 0.000E+000	5.130E-001
150	3.400E-001 2.411E-002	3.700E-001 0.000E+000	-.157E+001
100	1.626E-001 3.312E-002	1.912E-001 0.000E+000	-.863E+000
75	0.667E-002 1.079E-002	0.209E-002 0.000E+000	2.441E-001
60	5.630E-002 2.057E-002	5.749E-002 0.000E+000	-.577E-001
45	1.710E-002 1.172E-002	2.171E-002 0.000E+000	-.306E+000
30	0.792E-004 1.422E-003	9.095E-005 0.000E+000	5.409E-001
20	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY B	RANGE=124.5 M MEASURED B-1000	DEVIATION FROM MEAN
1338	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
800	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
700	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
600	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
512	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
510	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
450	2.709E-002 3.656E-003	2.902E-002 0.000E+000	-.255E+000
400	4.565E-002 8.062E-003	5.092E-002 0.000E+000	-.653E+000
300	4.784E-002 1.549E-002	4.418E-002 0.000E+000	2.363E-001
200	2.782E-001 1.589E-002	2.383E-001 0.000E+000	-.631E+000
150	3.164E-001 4.500E-002	2.851E-001 0.000E+000	6.938E-001
100	1.862E-001 4.822E-002	1.712E-001 0.000E+000	3.105E-001
75	1.167E-001 2.819E-002	1.081E-001 0.000E+000	3.046E-001
60	1.144E-001 4.451E-002	1.046E-001 0.000E+000	2.196E-001
45	1.546E-002 1.527E-002	2.772E-002 0.000E+000	7.690E-001
30	3.158E-003 3.207E-003	1.909E-004 0.000E+000	9.254E-001
20	1.362E-005 5.131E-005	1.551E-007 0.000E+000	2.664E-001
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY B	RANGE=166 M MEASURED B-1000	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
800	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
700	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
600	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
512	7.029E-005 3.050E-004	0.000E+000 0.000E+000	1.026E-001
510	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
450	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
400	1.621E-002 2.523E-003	1.787E-002 0.000E+000	-.659E+000
300	3.568E-002 7.004E-003	3.801E-002 0.000E+000	-.329E+000
200	4.356E-002 1.065E-002	4.848E-002 0.000E+000	-.462E+000
150	1.492E-001 1.392E-002	1.515E-001 0.000E+000	-.165E+000
100	2.427E-001 2.717E-002	2.927E-001 0.000E+000	-.184E+001
75	1.757E-001 3.365E-002	1.751E-001 0.000E+000	1.785E-002
60	1.276E-001 2.968E-002	1.235E-001 0.000E+000	1.369E-001
45	1.259E-001 3.786E-002	1.043E-001 0.000E+000	5.714E-001
30	5.949E-002 1.866E-002	5.009E-002 0.000E+000	5.037E-001
20	4.316E-003 2.371E-003	1.294E-003 0.000E+000	1.274E+000
10	2.702E-006 1.213E-005	0.402E-007 0.000E+000	1.535E-001
	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY B	RANGE=245 M MEASURED B-1000	DEVIATION FROM MEAN
1330	1.700E-005 9.310E-005	0.000E+000 0.000E+000	1.826E-001
1000	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
300	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
700	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
600	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
512	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
510	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
450	6.880E-003 1.804E-003	6.603E-003 0.000E+000	1.539E-001
400	1.753E-002 3.246E-003	1.295E-002 0.000E+000	1.413E+000
300	2.650E-002 6.524E-003	3.184E-002 0.000E+000	-0.818E+000
200	6.948E-002 1.188E-002	6.864E-002 0.000E+000	7.529E-002
150	1.379E-001 1.786E-002	1.557E-001 0.000E+000	-0.996E+000
100	1.111E-001 1.882E-002	1.139E-001 0.000E+000	-0.151E+000
75	9.930E-002 2.100E-002	6.826E-002 0.000E+000	1.478E+000
60	1.124E-001 2.495E-002	1.228E-001 0.000E+000	-0.417E+000
45	5.875E-002 1.447E-002	8.830E-002 0.000E+000	-0.284E+001
30	3.274E-003 2.193E-003	4.542E-003 0.000E+000	-0.578E+000
20	1.766E-005 6.446E-005	0.000E+000 0.000E+000	2.740E-001
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY B	RANGE=333 M MEASURED B-1000	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
800	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
700	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
600	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
512	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
510	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
450	2.635E-003 1.139E-003	2.579E-003 0.000E+000	4.861E-002
400	7.894E-003 1.909E-003	9.777E-003 0.000E+000	-.986E+000
300	1.487E-002 5.248E-003	1.486E-002 0.000E+000	1.796E-003
200	2.836E-002 6.912E-003	4.803E-002 0.000E+000	-.168E+001
150	6.886E-002 1.449E-002	6.518E-002 0.000E+000	2.542E-001
100	5.934E-002 1.883E-002	5.326E-002 0.000E+000	3.228E-001
75	4.878E-002 1.225E-002	5.817E-002 0.000E+000	-.773E+000
60	6.996E-002 2.829E-002	6.203E-002 0.000E+000	3.908E-001
45	4.881E-002 2.617E-002	3.507E-002 0.000E+000	2.191E-001
30	3.807E-003 2.540E-003	1.480E-003 0.000E+000	6.889E-001
20	6.250E-006 2.233E-005	0.000E+000 0.000E+000	2.798E-001
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY B	RANGE=398 M MEASURED B-1000	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
800	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
700	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
600	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
512	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
510	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
450	1.367E-003 5.974E-004	1.031E-003 0.000E+000	5.636E-001
400	4.655E-003 1.384E-003	4.203E-003 0.000E+000	3.265E-001
300	8.348E-003 3.524E-003	9.820E-003 0.000E+000	-4.17E+000
200	1.436E-002 4.205E-003	1.098E-002 0.000E+000	8.052E-001
150	3.815E-002 1.088E-002	6.736E-002 0.000E+000	-2.68E+001
100	3.382E-002 7.984E-003	4.849E-002 0.000E+000	-1.83E+001
75	3.121E-002 7.351E-003	3.295E-002 0.000E+000	-2.36E+000
60	4.344E-002 1.287E-002	4.858E-002 0.000E+000	-3.99E+000
45	2.521E-002 8.718E-003	2.492E-002 0.000E+000	3.324E-002
30	1.741E-003 1.764E-003	9.473E-004 0.000E+000	4.500E-001
20	1.353E-006 6.547E-006	0.000E+000 0.000E+000	2.127E-001
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY B	RANGE=41 M MEASURED 0-1000	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
800	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
700	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
600	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
512	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
510	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
450	6.460E-002 8.089E-003	5.730E-002 0.000E+000	9.024E-001
400	7.946E-002 1.104E-002	6.789E-002 0.000E+000	1.040E+000
300	4.009E-002 1.351E-002	4.299E-002 0.000E+000	-.214E+000
200	5.425E-001 2.573E-002	5.635E-001 0.000E+000	-.817E+000
150	2.726E-001 3.312E-002	2.615E-001 0.000E+000	-.269E+000
100	7.534E-002 2.373E-002	7.277E-002 0.000E+000	1.002E-001
75	3.223E-002 2.258E-002	2.256E-002 0.000E+000	4.270E-001
60	1.045E-002 1.145E-002	5.556E-003 0.000E+000	4.270E-001
45	2.061E-003 3.935E-003	0.000E+000 0.000E+000	5.237E-001
30	7.984E-005 3.336E-004	0.000E+000 0.000E+000	2.393E-001
20	3.091E-006 1.693E-005	0.000E+000 0.000E+000	1.826E-001
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000



GROUP	COUNTS/PHOTON LIBRARY C	RANGE=41 M MEASURED C-1000	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	1.832E-001 1.374E-002	1.871E-001 0.000E+000	-.288E+000
900	9.836E-002 8.860E-003	1.003E-001 0.000E+000	-.112E+001
700	3.375E-002 6.551E-003	4.460E-002 0.000E+000	-.165E+001
600	1.812E-002 6.183E-003	1.896E-002 0.000E+000	-.136E+000
512	2.563E-004 6.415E-004	1.830E-003 0.000E+000	-.245E+001
510	1.035E-002 5.629E-003	1.127E-002 0.000E+000	-.164E+000
450	8.699E-003 5.371E-003	7.585E-003 0.000E+000	2.075E-001
400	2.909E-002 1.129E-002	4.299E-002 0.000E+000	-.123E+001
300	3.156E-002 1.339E-002	2.208E-002 0.000E+000	7.096E-001
200	3.453E-001 2.111E-002	3.212E-001 0.000E+000	1.142E+000
150	2.209E-001 2.786E-002	2.009E-001 0.000E+000	4.305E-001
100	7.005E-002 2.735E-002	6.323E-002 0.000E+000	5.410E-001
75	2.900E-002 1.114E-002	2.934E-002 0.000E+000	2.348E-002
60	1.308E-002 1.010E-002	2.343E-002 0.000E+000	-.102E+001
45	3.434E-003 7.066E-003	1.328E-002 0.000E+000	-.139E+001
30	0.026E-005 3.266E-004	0.000E+000 0.000E+000	2.461E-001
15	2.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY C	RANGE=83 M MEASURED C-1000	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	1.371E-001 1.110E-002	1.477E-001 0.000E+000	-.953E+000
900	7.732E-002 8.754E-003	8.075E-002 0.000E+000	-.392E+000
700	3.231E-002 5.545E-003	3.662E-002 0.000E+000	-.776E+000
600	2.094E-002 4.603E-003	2.784E-002 0.000E+000	-.149E+001
512	2.727E-004 6.285E-004	0.000E+000 0.000E+000	4.338E-001
510	1.626E-002 7.641E-003	2.176E-002 0.000E+000	-.719E+000
450	1.457E-002 6.220E-003	1.290E-002 0.000E+000	2.682E-001
400	3.969E-002 1.364E-002	4.193E-002 0.000E+000	-.164E+000
300	5.082E-002 1.450E-002	4.199E-002 0.000E+000	6.058E-001
200	2.317E-001 1.688E-002	2.323E-001 0.000E+000	-.390E-001
150	2.645E-001 3.183E-002	2.600E-001 0.000E+000	1.421E-001
100	1.394E-001 2.097E-002	1.529E-001 0.000E+000	-.645E+000
75	7.180E-002 1.969E-002	7.685E-002 0.000E+000	-.256E+000
60	5.183E-002 1.855E-002	3.291E-002 0.000E+000	1.020E+000
45	1.548E-002 1.220E-002	3.753E-002 0.000E+000	-.100E+001
30	9.667E-004 1.685E-003	0.000E+000 0.000E+000	5.736E-001
20	5.104E-005 2.795E-004	0.000E+000 0.000E+000	1.026E-001
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY C	RANGE=124.5 M MEASURED C-1000	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	9.933E-002 7.370E-003	9.229E-002 0.000E+000	9.553E-001
800	5.596E-002 8.878E-003	6.102E-002 0.000E+000	-1.569E+000
700	3.070E-002 7.579E-003	2.533E-002 0.000E+000	7.188E-001
600	2.386E-002 6.207E-003	2.349E-002 0.000E+000	6.013E-002
512	5.799E-004 9.737E-004	2.128E-003 0.000E+000	-1.157E+001
510	1.770E-002 6.176E-003	1.383E-002 0.000E+000	6.396E-001
450	1.664E-002 6.094E-003	1.622E-002 0.000E+000	6.921E-002
400	4.167E-002 1.174E-002	4.088E-002 0.000E+000	6.706E-002
300	5.305E-002 1.400E-002	5.066E-002 0.000E+000	-1.400E+000
200	1.626E-001 1.850E-002	1.940E-001 0.000E+000	-1.169E+001
150	2.460E-001 3.201E-002	2.759E-001 0.000E+000	-1.870E+000
120	1.499E-001 2.663E-002	1.711E-001 0.000E+000	-1.795E+000
75	1.000E-001 2.601E-002	1.146E-001 0.000E+000	-1.530E+000
60	9.419E-002 3.453E-002	9.261E-002 0.000E+000	4.586E-002
45	3.070E-002 1.313E-002	4.434E-002 0.000E+000	-1.420E+000
30	2.395E-003 2.160E-003	5.106E-003 0.000E+000	-1.124E+001
20	1.401E-007 4.565E-007	1.517E-007 0.000E+000	-1.255E-001
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY C	RANGE=166 M MEASURED C-1000	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	7.190E-002 7.289E-003	7.069E-002 0.000E+000	1.668E-001
800	4.356E-002 5.481E-003	4.696E-002 0.000E+000	-.629E+000
700	2.719E-002 6.696E-003	3.589E-002 0.000E+000	-.130E+001
600	2.236E-002 6.055E-003	1.770E-002 0.000E+000	7.697E-001
512	7.361E-004 1.485E-003	0.000E+000 0.000E+000	4.956E-001
510	1.666E-002 7.107E-003	2.527E-002 0.000E+000	-.121E+001
450	2.031E-002 5.719E-003	3.177E-002 0.000E+000	-.280E+001
400	4.170E-002 1.154E-002	4.527E-002 0.000E+000	-.389E+000
300	5.991E-002 1.711E-002	3.989E-002 0.000E+000	1.217E+000
200	1.241E-001 1.290E-002	1.190E-001 0.000E+000	4.017E-001
150	1.947E-001 2.672E-002	1.682E-001 0.000E+000	9.893E-001
100	1.476E-001 3.141E-002	1.561E-001 0.000E+000	-.269E+000
75	1.029E-001 2.279E-002	7.308E-002 0.000E+000	1.308E+000
60	1.060E-001 2.494E-002	1.234E-001 0.000E+000	-.664E+000
45	4.597E-002 1.874E-002	5.745E-002 0.000E+000	-.612E+000
30	4.412E-003 4.847E-003	1.431E-003 0.000E+000	6.150E-001
20	4.045E-006 1.304E-005	0.000E+000 0.000E+000	3.102E-001
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY C	RANGE=245 M MEASURED C-1000	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	3.846E-002 4.757E-003	2.761E-002 0.000E+000	2.285E+000
800	2.490E-002 4.537E-003	2.942E-002 0.000E+000	-.958E+000
700	1.827E-002 4.092E-003	1.667E-002 0.000E+000	3.904E-001
600	1.655E-002 3.388E-003	1.595E-002 0.000E+000	1.775E-001
512	5.250E-004 1.034E-003	0.000E+000 0.000E+000	5.087E-001
510	1.447E-002 4.785E-003	1.789E-002 0.000E+000	-.715E+000
450	1.307E-002 5.404E-003	1.651E-002 0.000E+000	-.637E+000
400	3.312E-002 9.805E-003	3.382E-002 0.000E+000	-.711E-001
300	4.237E-002 1.702E-002	4.730E-002 0.000E+000	-.289E+000
200	6.576E-002 1.483E-002	1.005E-001 0.000E+000	-.234E+001
150	1.217E-001 2.041E-002	1.212E-001 0.000E+000	2.424E-002
100	9.531E-002 2.099E-002	9.574E-002 0.000E+000	4.558E-001
75	8.131E-002 1.836E-002	9.309E-002 0.000E+000	-.641E+000
50	9.145E-002 2.532E-002	1.065E-001 0.000E+000	-.594E+000
45	4.720E-002 2.249E-002	2.791E-002 0.000E+000	8.581E-001
30	3.865E-003 2.407E-003	5.713E-003 0.000E+000	-.110E+001
20	7.618E-005 2.751E-004	0.000E+000 0.000E+000	2.770E-001
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY C	RANGE=333 M MEASURED C-1000	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1200	1.979E-002 2.609E-003	1.561E-002 0.000E+000	1.602E+000
800	1.271E-002 2.723E-003	9.311E-003 0.000E+000	1.249E+000
700	1.190E-002 3.181E-003	1.270E-002 0.000E+000	-.251E+000
600	1.181E-002 2.934E-003	9.182E-003 0.000E+000	8.940E-001
512	2.347E-004 5.322E-004	2.049E-004 0.000E+000	5.600E-002
510	9.407E-003 3.681E-003	6.350E-003 0.000E+000	8.305E-001
450	8.504E-003 4.224E-003	6.116E-003 0.000E+000	5.651E-001
400	2.312E-002 7.014E-003	1.882E-002 0.000E+000	6.141E-001
300	3.213E-002 8.920E-003	4.631E-002 0.000E+000	-.159E+001
200	3.632E-002 1.036E-002	3.927E-002 0.000E+000	-.284E+000
150	7.143E-002 1.523E-002	8.200E-002 0.000E+000	-.693E+000
100	6.537E-002 2.381E-002	5.217E-002 0.000E+000	5.546E-001
75	5.410E-002 1.952E-002	3.658E-002 0.000E+000	8.971E-001
60	6.494E-002 2.269E-002	7.672E-002 0.000E+000	-.519E+000
45	3.618E-002 1.085E-002	4.848E-002 0.000E+000	-.396E+000
30	2.014E-003 1.616E-003	2.550E-003 0.000E+000	-.332E+000
20	8.960E-006 4.184E-005	0.000E+000 0.000E+000	2.142E-001
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY C	RANGE=390 M MEASURED C-1000	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	1.195E-002 1.352E-003	1.242E-002 0.000E+000	- .344E+000
800	9.126E-003 1.874E-003	1.300E-002 0.000E+000	- .206E+001
700	7.472E-003 2.806E-003	9.352E-003 0.000E+000	- .651E+000
600	8.631E-003 3.169E-003	7.453E-003 0.000E+000	3.717E-001
512	3.052E-004 4.752E-004	2.108E-005 0.000E+000	5.979E-001
510	7.473E-003 2.430E-003	9.721E-003 0.000E+000	- .924E+000
450	7.598E-003 2.481E-003	9.041E-003 0.000E+000	- .581E+000
400	1.486E-002 4.576E-003	1.319E-002 0.000E+000	3.644E-001
300	2.135E-002 4.750E-003	1.752E-002 0.000E+000	8.075E-001
200	2.109E-002 4.648E-003	1.364E-002 0.000E+000	1.603E+000
150	4.690E-002 1.020E-002	5.671E-002 0.000E+000	- .962E+000
100	3.816E-002 1.012E-002	4.068E-002 0.000E+000	- .249E+000
75	3.300E-002 9.596E-003	3.257E-002 0.000E+000	4.493E-002
60	4.394E-002 1.245E-002	4.402E-002 0.000E+000	- .681E-002
45	2.482E-002 8.229E-003	2.390E-002 0.000E+000	1.117E-001
30	1.676E-003 1.602E-003	3.859E-004 0.000E+000	8.056E-001
20	4.389E-006 1.960E-005	0.000E+000 0.000E+000	2.239E-001
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY A	RANGE=20, M MEASURED A-100	DEVIATION FROM MEAN
1330	0.000E+000	0.000E+000	0.000E+000
	0.000E+000	0.000E+000	0.000E+000
1000	5.940E-002	4.660E-002	9.383E+000
	1.365E-003	0.000E+000	
800	3.043E-002	1.541E-002	1.318E+001
	1.140E-003	0.000E+000	
700	2.947E-001	2.520E-001	1.297E+001
	7.253E-003	0.000E+000	
600	1.161E-002	0.000E+000	1.064E+001
	1.092E-003	0.000E+000	
512	1.011E-001	1.539E-001	1.835E+000
	1.479E-002	0.000E+000	
510	1.066E-002	0.000E+000	1.640E+001
	1.130E-003	0.000E+000	
450	2.472E-002	5.716E-003	1.323E+001
	1.437E-003	0.000E+000	
400	7.060E-002	1.087E-001	-1.151E+002
	2.504E-003	0.000E+000	
300	6.154E-002	7.729E-003	1.950E+001
	2.760E-003	0.000E+000	
200	1.183E-001	1.491E-001	-9.957E+001
	3.221E-003	0.000E+000	
150	9.520E-002	1.094E-001	-4.470E+001
	2.989E-003	0.000E+000	
120	1.492E-002	2.235E-002	-4.466E+001
	1.593E-003	0.000E+000	
75	2.003E-002	1.309E-002	7.767E+000
	9.971E-004	0.000E+000	
60	2.531E-002	3.929E-002	-1.104E+002
	1.339E-003	0.000E+000	
45	1.524E-002	3.507E-003	1.007E+001
	1.164E-003	0.000E+000	
30	3.047E-004	0.000E+000	2.359E+000
	1.652E-004	0.000E+000	
20	0.000E+000	0.000E+000	0.000E+000
	0.000E+000	0.000E+000	
10	0.000E+000	0.000E+000	0.000E+000
	0.000E+000	0.000E+000	



GROUP	COUNTS/PHOTON LIBRARY A	RANGE=41 M MEASURED A-100	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	5.024E-002 1.441E-003	5.126E-002 0.000E+000	-.706E+000
800	2.684E-002 1.037E-003	1.473E-002 0.000E+000	1.167E+001
700	2.449E-001 2.425E-003	2.475E-001 0.000E+000	-.100E+001
600	2.253E-002 1.454E-003	4.448E-002 0.000E+000	-.150E+002
512	1.437E-001 1.051E-003	1.149E-001 0.000E+000	1.553E+001
510	3.117E-002 1.474E-003	5.312E-002 0.000E+000	-.149E+002
450	3.306E-002 1.450E-003	5.969E-002 0.000E+000	-.182E+002
400	0.656E-002 2.770E-003	7.904E-002 0.000E+000	2.707E+000
300	9.430E-002 2.961E-003	4.971E-002 0.000E+000	1.509E+001
200	1.205E-001 3.251E-003	1.407E-001 0.000E+000	-.621E+001
150	1.200E-001 3.963E-003	1.233E-001 0.000E+000	-.838E+000
100	4.043E-002 2.919E-003	1.128E-002 0.000E+000	9.984E+000
75	3.013E-002 1.998E-003	5.140E-003 0.000E+000	1.250E+001
60	2.934E-002 1.945E-003	0.000E+000 0.000E+000	1.509E+001
45	1.580E-002 1.109E-003	0.000E+000 0.000E+000	1.424E+001
30	7.779E-004 1.498E-004	0.000E+000 0.000E+000	5.192E+000
20	1.608E-005 1.227E-005	0.000E+000 0.000E+000	1.311E+000
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY A	RANGE=83 M MEASURED A-100	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	3.550E-002 9.961E-004	2.665E-002 0.000E+000	8.962E+000
800	2.122E-002 8.789E-004	5.235E-002 0.000E+000	-3.354E+002
700	1.677E-001 2.040E-003	1.022E-001 0.000E+000	3.212E+001
600	3.517E-002 1.045E-003	4.392E-002 0.000E+000	-4.473E+001
512	8.760E-002 1.424E-003	1.565E-001 0.000E+000	-4.403E+002
510	3.923E-002 1.714E-003	4.359E-002 0.000E+000	-2.254E+001
450	4.000E-002 1.097E-003	6.780E-003 0.000E+000	1.752E+001
400	1.021E-001 3.537E-003	0.600E-002 0.000E+000	4.531E+000
300	1.260E-001 3.051E-003	9.216E-002 0.000E+000	8.790E+000
200	1.190E-001 3.652E-003	2.150E-001 0.000E+000	-2.260E+002
150	1.644E-001 4.377E-003	1.720E-001 0.000E+000	-1.175E+001
100	7.309E-002 3.028E-003	0.000E+000 0.000E+000	1.909E+001
75	4.653E-002 2.598E-003	4.165E-002 0.000E+000	1.000E+000
60	4.500E-002 2.780E-003	0.000E+000 0.000E+000	1.647E+001
45	2.174E-002 2.126E-003	0.000E+000 0.000E+000	6.041E+000
30	1.177E-003 2.379E-004	0.000E+000 0.000E+000	1.234E+000
20	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY A	RANGE=124.5 M MEASURED A-100	DEVIATION FROM MEAN
1338	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	2.633E-002 8.987E-004	2.414E-002 0.000E+000	2.443E+000
800	1.573E-002 6.613E-004	8.935E-003 0.000E+000	1.028E+001
700	1.181E-001 1.869E-003	1.014E-001 0.000E+000	8.953E+000
600	3.285E-002 1.219E-003	3.218E-002 0.000E+000	5.444E-001
512	5.572E-002 8.090E-004	5.375E-002 0.000E+000	2.435E+000
510	3.796E-002 1.477E-003	4.480E-002 0.000E+000	-.462E+001
450	3.674E-002 1.415E-003	3.895E-002 0.000E+000	4.891E+000
400	9.565E-002 2.786E-003	6.960E-002 0.000E+000	9.352E+000
300	1.350E-001 4.857E-003	1.301E-001 0.000E+000	1.189E+000
200	1.163E-001 3.652E-003	1.288E-001 0.000E+000	-.341E+001
150	1.704E-001 6.046E-003	3.962E-002 0.000E+000	2.163E+001
100	1.882E-001 5.513E-003	1.321E-001 0.000E+000	-.433E+001
75	7.551E-002 3.734E-003	1.101E-001 0.000E+000	-.925E+001
60	6.651E-002 4.221E-003	6.523E-002 0.000E+000	3.831E-001
45	3.209E-002 2.477E-003	1.854E-002 0.000E+000	5.473E+000
30	2.836E-003 3.965E-004	2.638E-005 0.000E+000	5.868E+000
20	3.814E-006 2.718E-006	0.000E+000 0.000E+000	1.483E+000
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY A	RANGE=166 M MEASURED A-100	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	1.914E-002 8.006E-004	1.678E-002 0.000E+000	2.910E+000
800	1.127E-002 7.047E-004	7.346E-003 0.000E+000	5.567E+000
700	8.123E-002 1.238E-003	7.357E-002 0.000E+000	6.187E+000
600	2.880E-002 1.053E-003	3.546E-002 0.000E+000	-.632E+001
512	3.555E-002 6.027E-004	6.916E-002 0.000E+000	-.557E+002
510	3.514E-002 1.002E-003	1.774E-002 0.000E+000	1.737E+001
450	3.432E-002 1.305E-003	1.701E-002 0.000E+000	1.327E+001
400	8.016E-002 2.741E-003	1.230E-001 0.000E+000	-.127E+002
300	1.234E-001 3.890E-003	1.167E-001 0.000E+000	1.714E+000
200	9.955E-002 3.943E-003	5.716E-002 0.000E+000	1.075E+001
150	1.635E-001 4.707E-003	1.920E-001 0.000E+000	-.595E+001
100	1.172E-001 5.976E-003	3.140E-002 0.000E+000	1.435E+001
75	7.679E-002 3.669E-003	2.822E-002 0.000E+000	1.323E+001
60	8.765E-002 5.365E-003	5.448E-002 0.000E+000	6.183E+000
45	4.260E-002 3.529E-003	1.156E-002 0.000E+000	8.797E+000
30	2.912E-003 3.745E-004	8.799E-003 0.000E+000	-.157E+002
20	1.739E-006 9.931E-007	0.000E+000 0.000E+000	1.751E+000
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS PER BIN LIBRARY A	MEASURED A-100	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	1.074E-002 3.929E-004	1.098E-002 0.000E+000	- .609E+000
800	6.137E-003 3.696E-004	9.921E-003 0.000E+000	- .102E+002
700	4.007E-002 9.661E-004	3.466E-002 0.000E+000	5.595E+000
600	2.041E-002 8.336E-004	3.192E-002 0.000E+000	- .138E+002
512	1.438E-002 3.573E-004	7.156E-003 0.000E+000	2.020E+001
510	2.352E-002 8.902E-004	1.081E-002 0.000E+000	1.427E+001
450	2.371E-002 8.396E-004	1.774E-002 0.000E+000	7.106E+000
400	6.089E-002 2.239E-003	1.382E-002 0.000E+000	1.656E+001
300	9.273E-002 3.549E-003	4.366E-002 0.000E+000	1.302E+001
200	.402E-002 1.851E-003	7.793E-002 0.000E+000	- .702E+001
150	1.292E-001 4.168E-003	7.613E-002 0.000E+000	1.274E+001
100	9.084E-002 3.568E-003	1.002E-001 0.000E+000	- .262E+001
75	7.496E-002 7.514E-003	2.235E-002 0.000E+000	7.001E+000
60	8.839E-002 5.567E-003	2.519E-003 0.000E+000	1.543E+001
45	4.720E-002 2.685E-003	3.747E-002 0.000E+000	3.623E+000
30	4.174E-003 9.346E-004	5.717E-005 0.000E+000	4.405E+000
20	1.234E-006 7.773E-007	0.000E+000 0.000E+000	1.587E+000
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY A	RANGE=333 M MEASURED A-100	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	5.408E-003 2.168E-004	7.323E-003 0.000E+000	-0.846E+001
800	3.503E-003 2.155E-004	4.636E-003 0.000E+000	-0.525E+001
700	1.985E-002 4.154E-004	2.539E-002 0.000E+000	-0.133E+002
600	1.132E-002 4.624E-004	5.323E-003 0.000E+000	1.296E+001
512	5.400E-003 2.491E-004	6.302E-003 0.000E+000	-0.330E+001
510	1.485E-002 5.977E-004	2.442E-003 0.000E+000	2.076E+001
450	1.346E-002 6.769E-004	6.068E-003 0.000E+000	1.091E+001
400	3.959E-002 9.671E-004	1.445E-002 0.000E+000	2.600E+001
300	6.300E-002 5.677E-003	7.057E-002 0.000E+000	-0.250E+001
200	4.240E-002 1.006E-003	3.137E-002 0.000E+000	6.153E+000
150	8.336E-002 3.102E-003	9.168E-002 0.000E+000	-0.260E+001
100	6.455E-002 2.471E-003	7.099E-002 0.000E+000	-0.504E+001
75	6.147E-002 4.201E-003	5.174E-002 0.000E+000	2.315E+000
60	6.400E-002 3.756E-003	1.090E-001 0.000E+000	-0.119E+002
45	3.106E-002 2.177E-003	1.011E-002 0.000E+000	9.993E+000
30	2.376E-003 4.504E-004	7.063E-004 0.000E+000	3.530E+000
20	5.473E-006 3.306E-006	0.000E+000 0.000E+000	1.656E+000
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY B	RANGE=20. M MEASURED B-100	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
800	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
700	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
600	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
512	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
510	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
450	7.940E-002 1.832E-003	7.570E-002 0.000E+000	1.901E+000
400	9.463E-002 1.930E-003	4.334E-002 0.000E+000	2.657E+001
300	2.445E-002 2.052E-003	0.000E+000 0.000E+000	1.192E+001
200	6.606E-001 3.061E-003	6.261E-001 0.000E+000	1.126E+001
150	1.759E-001 7.626E-003	1.593E-001 0.000E+000	2.175E+000
100	3.174E-002 2.743E-003	2.585E-002 0.000E+000	2.147E+000
75	4.079E-003 1.239E-003	0.000E+000 0.000E+000	3.937E+000
50	1.650E-003 2.097E-004	0.000E+000 0.000E+000	2.051E+000
45	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
32	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
20	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY B	RANGE=41 M MEASURED B-100	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
800	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
700	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
600	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
512	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
510	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
450	6.460E-002 1.477E-003	1.343E-002 0.000E+000	3.465E+001
400	7.946E-002 2.016E-003	5.296E-002 0.000E+000	1.315E+001
300	4.009E-002 2.467E-003	1.048E-003 0.000E+000	1.550E+001
200	5.425E-001 4.698E-003	5.267E-001 0.000E+000	3.360E+000
150	2.726E-001 6.047E-003	5.325E-001 0.000E+000	-.429E+002
100	7.534E-002 4.332E-003	1.057E-001 0.000E+000	-.700E+001
75	3.223E-002 4.123E-003	0.000E+000 0.000E+000	7.015E+000
60	1.045E-002 2.090E-003	0.000E+000 0.000E+000	4.997E+000
45	2.061E-003 7.184E-004	0.000E+000 0.000E+000	2.860E+000
30	7.984E-005 6.090E-005	0.000E+000 0.000E+000	1.311E+000
20	3.091E-006 3.091E-006	0.000E+000 0.000E+000	1.000E+000
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000



COUNTS/PHOTON GROUP	LIBRARY B	RANGE=83 M MEASURED B-100	DEVIATION FROM MEAN
1350	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
800	1.651E-005 1.651E-005	0.000E+000 0.000E+000	1.000E+000
700	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
600	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
512	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
510	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
450	4.199E-002 1.110E-003	3.922E-002 0.000E+000	2.497E+000
400	6.441E-002 1.438E-003	7.507E-002 0.000E+000	-7.41E+001
300	4.505E-002 2.153E-003	3.570E-002 0.000E+000	4.342E+000
200	3.559E-001 3.248E-003	3.068E-001 0.000E+000	1.511E+001
150	3.408E-001 4.401E-003	1.847E-001 0.000E+000	3.546E+001
100	1.626E-001 6.047E-003	9.101E-002 0.000E+000	1.183E+001
75	8.057E-002 7.431E-003	7.339E-002 0.000E+000	3.871E+000
50	5.520E-002 3.755E-003	0.000E+000 0.000E+000	1.499E+001
40	1.718E-002 2.140E-003	0.000E+000 0.000E+000	8.029E+000
30	6.792E-004 2.596E-004	0.000E+000 0.000E+000	3.387E+000
20	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY B	RANGE=124.5 M MEASURED B-100	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
800	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
700	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
600	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
512	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
510	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
450	2.709E-002 6.676E-004	4.386E-002 0.000E+000	-.251E+002
400	4.565E-002 1.472E-003	5.498E-002 0.000E+000	-.633E+001
300	4.784E-002 2.829E-003	6.816E-002 0.000E+000	-.718E+001
200	2.282E-001 2.900E-003	3.025E-001 0.000E+000	-.255E+002
150	3.164E-001 8.215E-003	2.746E-001 0.000E+000	5.002E+000
100	1.862E-001 8.803E-003	1.824E-001 0.000E+000	4.282E-001
75	1.167E-001 5.147E-003	5.543E-002 0.000E+000	1.189E+001
60	1.144E-001 8.127E-003	5.009E-002 0.000E+000	7.909E+000
45	3.946E-002 2.707E-003	0.000E+000 0.000E+000	1.416E+001
30	3.158E-003 5.854E-004	5.358E-004 0.000E+000	4.479E+000
20	1.382E-005 9.368E-006	0.000E+000 0.000E+000	1.476E+000
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY B	RANGE=166 M MEASURED B-100	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
800	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
700	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
600	7.029E-005 7.029E-005	0.000E+000 0.000E+000	1.000E+000
517	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
510	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
450	1.621E-002 4.606E-004	0.000E+000 0.000E+000	-0.274E+002
400	3.560E-002 1.293E-003	0.000E+000 0.000E+000	2.513E+001
300	4.356E-002 1.944E-003	0.000E+000 0.000E+000	1.643E+000
200	1.492E-001 2.542E-003	0.000E+000 0.000E+000	3.450E+000
150	2.427E-001 4.961E-003	0.000E+000 0.000E+000	-0.283E+001
100	1.757E-001 5.144E-003	0.000E+000 0.000E+000	-0.217E+002
75	1.276E-001 5.420E-003	0.000E+000 0.000E+000	-0.137E+002
60	1.259E-001 5.711E-003	0.000E+000 0.000E+000	1.001E+001
45	5.949E-002 3.407E-003	0.000E+000 0.000E+000	6.419E+000
30	4.316E-003 4.330E-004	0.000E+000 0.000E+000	6.441E+000
20	0.702E-006 2.214E-006	0.000E+000 0.000E+000	1.220E+000
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY B	RANGE=245 M MEASURED B-100	DEVIATION FROM MEAN
1330	1.700E-005	0.000E+000	1.000E+000
	1.700E-005	0.000E+000	
1000	0.000E+000	0.000E+000	0.000E+000
	0.000E+000	0.000E+000	
800	0.000E+000	0.000E+000	0.000E+000
	0.000E+000	0.000E+000	
700	0.000E+000	0.000E+000	0.000E+000
	0.000E+000	0.000E+000	
600	0.000E+000	0.000E+000	0.000E+000
	0.000E+000	0.000E+000	
512	0.000E+000	0.000E+000	0.000E+000
	0.000E+000	0.000E+000	
510	0.000E+000	0.000E+000	0.000E+000
	0.000E+000	0.000E+000	
450	6.800E-003	6.317E-003	1.710E+000
	3.294E-004	0.000E+000	
400	1.753E-002	9.097E-003	1.424E+001
	5.926E-004	0.000E+000	
300	2.650E-002	2.176E-002	3.980E+000
	1.191E-003	0.000E+000	
200	6.948E-002	5.608E-002	6.626E+000
	2.023E-003	0.000E+000	
150	1.379E-001	6.524E-002	2.229E+001
	3.260E-003	0.000E+000	
100	1.111E-001	1.733E-001	-.182E+002
	3.399E-003	0.000E+000	
75	9.930E-002	8.060E-002	2.791E+000
	3.835E-003	0.000E+000	
50	1.124E-001	3.062E-001	-.425E+002
	4.555E-003	0.000E+000	
45	5.875E-002	7.503E-002	-.616E+001
	2.641E-003	0.000E+000	
30	3.274E-003	3.304E-004	7.352E+000
	4.004E-004	0.000E+000	
20	1.766E-005	0.000E+000	1.501E+000
	1.177E-005	0.000E+000	
10	0.000E+000	0.000E+000	0.000E+000
	0.000E+000	0.000E+000	

GROUP	COUNTS/PHOTON LIBRARY B	RANGE=333 M MEASURED B-100	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
800	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
700	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
600	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
512	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
510	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
450	2.635E-003 2.000E-004	0.000E+000 0.000E+000	1.267E+001
400	7.074E-003 3.405E-004	0.000E+000 0.000E+000	2.265E+001
300	1.487E-002 9.501E-004	1.720E-002 0.000E+000	-1.251E+001
200	2.036E-002 1.262E-003	2.765E-002 0.000E+000	5.602E-001
150	6.006E-002 2.645E-003	4.971E-002 0.000E+000	7.242E+000
100	5.934E-002 3.430E-003	9.075E-002 0.000E+000	-1.114E+002
75	4.070E-002 2.237E-003	6.500E-002 0.000E+000	-1.732E+001
60	6.996E-002 3.705E-003	9.761E-002 0.000E+000	-1.746E+001
45	4.001E-002 4.777E-003	6.175E-002 0.000E+000	-1.430E+001
30	3.007E-003 4.630E-004	5.016E-005 0.000E+000	6.374E+000
20	6.250E-006 4.070E-006	0.000E+000 0.000E+000	1.533E+000
13	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY C	RANGE=20. M MEASURED C-100	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	2.205E-001 2.646E-003	2.406E-001 0.000E+000	-.760E+001
800	1.121E-001 1.779E-003	1.116E-001 0.000E+000	2.008E-001
700	3.011E-002 1.016E-003	2.685E-002 0.000E+000	3.211E+000
600	8.795E-003 8.631E-004	7.846E-003 0.000E+000	1.899E+000
512	4.417E-004 1.902E-004	0.000E+000 0.000E+000	2.323E+000
510	6.838E-003 7.795E-004	8.279E-003 0.000E+000	-.184E+001
450	5.911E-003 6.914E-004	0.000E+000 0.000E+000	8.550E+000
400	1.296E-002 1.474E-003	7.023E-002 0.000E+000	-.388E+002
300	2.110E-002 1.640E-003	0.000E+000 0.000E+000	1.286E+001
200	4.160E-001 3.318E-003	4.031E-001 0.000E+000	4.133E+000
150	1.536E-001 3.963E-003	1.009E-001 0.000E+000	1.332E+001
100	4.689E-002 2.241E-003	0.000E+000 0.000E+000	2.092E+001
75	8.653E-003 1.316E-003	2.810E-002 0.000E+000	-.147E+002
60	2.327E-003 9.576E-004	0.000E+000 0.000E+000	2.430E+000
45	3.638E-004 2.967E-004	0.000E+000 0.000E+000	1.226E+000
30	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
20	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY C	RANGE=41 M MEASURED C-100	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	1.832E-001 2.509E-003	1.409E-001 0.000E+000	1.686E+001
800	9.836E-002 1.618E-003	8.625E-002 0.000E+000	7.484E+000
700	3.375E-002 1.196E-003	2.502E-002 0.000E+000	7.301E+000
600	1.812E-002 1.129E-003	2.134E-002 0.000E+000	-.285E+001
512	2.563E-004 1.171E-004	0.000E+000 0.000E+000	2.188E+000
510	1.035E-002 1.028E-003	1.499E-002 0.000E+000	-.451E+001
450	8.699E-003 9.805E-004	2.310E-002 0.000E+000	-.146E+002
400	2.909E-002 2.061E-003	1.906E-002 0.000E+000	4.867E+000
300	3.158E-002 2.444E-003	0.000E+000 0.000E+000	1.292E+001
200	3.453E-001 3.855E-003	3.712E-001 0.000E+000	-.119E+002
150	2.209E-001 5.086E-003	2.041E-001 0.000E+000	3.289E+000
100	7.805E-002 4.974E-003	7.100E-002 0.000E+000	1.411E+000
75	2.980E-002 2.034E-003	0.000E+000 0.000E+000	1.465E+001
60	1.308E-002 1.844E-003	0.000E+000 0.000E+000	7.091E+000
45	3.434E-003 1.290E-003	0.000E+000 0.000E+000	2.662E+000
30	8.076E-005 5.762E-005	0.000E+000 0.000E+000	1.348E+000
20	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY C	RANGE=83 M MEASURED C-100	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	1.371E-001 2.026E-003	1.016E-001 0.000E+000	1.752E+001
800	7.732E-002 1.598E-003	7.477E-002 0.000E+000	1.592E+000
700	3.231E-002 1.012E-003	3.946E-002 0.000E+000	-.706E+001
600	2.094E-002 0.404E-004	2.071E-002 0.000E+000	-.924E+001
512	2.727E-004 1.140E-004	0.000E+000 0.000E+000	2.376E+000
510	1.626E-002 1.395E-003	2.347E-002 0.000E+000	-.517E+001
450	1.457E-002 1.136E-003	0.000E+000 0.000E+000	1.203E+001
400	3.969E-002 2.491E-003	5.419E-003 0.000E+000	1.376E+001
300	5.002E-002 2.662E-003	7.799E-002 0.000E+000	-.102E+002
200	2.317E-001 3.002E-003	2.053E-001 0.000E+000	-.173E+002
150	2.645E-001 5.011E-003	2.247E-001 0.000E+000	6.040E+000
100	1.394E-001 3.020E-003	1.335E-001 0.000E+000	1.536E+000
75	7.100E-002 3.596E-003	1.002E-001 0.000E+000	-.301E+002
60	5.103E-002 3.307E-003	0.000E+000 0.000E+000	1.530E+001
45	1.540E-002 2.227E-003	0.000E+000 0.000E+000	6.951E+000
30	9.667E-004 3.077E-004	0.000E+000 0.000E+000	3.142E+000
20	5.104E-005 5.104E-005	0.000E+000 0.000E+000	1.000E+000
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000



GROUP	COUNTS/PHOTON LIBRARY C	RANGE=124.5 M MEASURED C-100	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	9.933E-002 1.346E-003	9.438E-002 0.000E+000	3.677E+000
800	5.596E-002 1.621E-003	4.833E-002 0.000E+000	4.704E+000
700	3.078E-002 1.384E-003	2.036E-002 0.000E+000	7.533E+000
600	2.386E-002 1.133E-003	1.574E-003 0.000E+000	1.967E+001
512	5.999E-004 1.778E-004	0.000E+000 0.000E+000	3.375E+000
510	1.778E-002 1.128E-003	5.567E-002 0.000E+000	-1.336E+002
450	1.664E-002 1.113E-003	3.428E-002 0.000E+000	-1.158E+002
400	4.167E-002 2.143E-003	2.396E-002 0.000E+000	8.265E+000
300	5.305E-002 2.556E-003	4.756E-002 0.000E+000	2.150E+000
200	1.626E-001 3.378E-003	3.247E-001 0.000E+000	-1.480E+002
150	2.480E-001 5.844E-003	1.687E-001 0.000E+000	1.357E+001
100	1.499E-001 4.862E-003	4.121E-002 0.000E+000	2.235E+001
75	1.208E-001 4.749E-003	2.625E-001 0.000E+000	-1.340E+002
60	9.419E-002 6.303E-003	1.826E-001 0.000E+000	-1.140E+002
45	3.878E-002 2.416E-003	7.645E-002 0.000E+000	-1.155E+002
30	2.395E-003 3.981E-004	0.000E+000 0.000E+000	6.017E+000
20	1.401E-007 8.334E-008	0.000E+000 0.000E+000	1.680E+000
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY C	RANGE=166 M MEASURED C-100	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	7.190E-002 1.331E-003	1.257E-001 0.000E+000	-.404E+002
800	4.356E-002 9.861E-004	4.579E-002 0.000E+000	-.226E+001
700	2.719E-002 1.222E-003	3.441E-002 0.000E+000	-.590E+001
600	2.236E-002 1.106E-003	2.372E-002 0.000E+000	-.123E+001
512	7.361E-004 2.712E-004	0.000E+000 0.000E+000	2.714E+000
510	1.666E-002 1.298E-003	1.004E-002 0.000E+000	5.101E+000
450	2.031E-002 1.044E-003	1.694E-002 0.000E+000	3.234E+000
400	4.170E-002 2.106E-003	0.915E-002 0.000E+000	-.225E+002
300	5.991E-002 3.124E-003	1.263E-002 0.000E+000	1.514E+001
200	1.241E-001 2.356E-003	1.532E-001 0.000E+000	-.123E+002
150	1.947E-001 4.879E-003	1.395E-001 0.000E+000	1.132E+001
100	1.476E-001 5.734E-003	6.355E-002 0.000E+000	1.466E+001
75	1.029E-001 4.161E-003	2.407E-002 0.000E+000	1.094E+001
60	1.060E-001 4.553E-003	1.314E-001 0.000E+000	-.539E+001
45	4.597E-002 3.421E-003	1.707E-002 0.000E+000	0.215E+000
30	4.412E-003 0.850E-004	1.192E-003 0.000E+000	3.639E+000
20	4.045E-006 2.301E-006	0.000E+000 0.000E+000	1.699E+000
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY C	RANGE=245 M MEASURED C-100	DEVIATION FROM MEAN
1330	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	3.848E-002 8.685E-004	2.734E-002 0.000E+000	1.283E+001
800	2.498E-002 8.465E-004	7.955E-003 0.000E+000	2.011E+001
700	1.827E-002 7.471E-004	2.748E-002 0.000E+000	-.123E+002
600	1.655E-002 6.185E-004	8.567E-003 0.000E+000	1.291E+001
512	5.258E-004 1.887E-004	0.000E+000 0.000E+000	2.786E+000
510	1.447E-002 8.736E-004	7.134E-003 0.000E+000	8.396E+000
450	1.307E-002 9.866E-004	3.623E-002 0.000E+000	-.234E+002
400	3.312E-002 1.790E-003	4.125E-003 0.000E+000	1.620E+001
300	4.237E-002 3.107E-003	2.890E-002 0.000E+000	4.337E+000
200	6.576E-002 2.708E-003	4.589E-002 0.000E+000	7.339E+000
150	1.217E-001 3.727E-003	1.277E-001 0.000E+000	-.159E+001
100	9.531E-002 3.832E-003	5.902E-002 0.000E+000	9.469E+000
75	8.131E-002 3.353E-003	5.812E-002 0.000E+000	6.915E+000
60	7.145E-002 4.622E-003	4.553E-002 0.000E+000	9.935E+000
45	4.720E-002 4.106E-003	3.155E-002 0.000E+000	3.812E+000
30	3.065E-003 4.394E-004	5.774E-004 0.000E+000	5.662E+000
20	7.018E-005 5.022E-005	0.000E+000 0.000E+000	1.517E+000
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

GROUP	COUNTS/PHOTON LIBRARY C	RANGE=333 M MEASURED C-100	DEVIATION FROM MEAN
1350	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000
1000	1.979E-002 4.764E-004	2.165E-002 0.000E+000	-.390E+001
800	1.271E-002 4.972E-004	1.812E-002 0.000E+000	-.100E+002
700	1.190E-002 5.807E-004	7.480E-003 0.000E+000	7.609E+000
600	1.181E-002 5.357E-004	0.000E+000 0.000E+000	2.204E+001
512	2.347E-004 9.716E-005	0.000E+000 0.000E+000	2.416E+000
510	9.407E-003 6.721E-004	9.585E-003 0.000E+000	-.265E+000
450	8.504E-003 7.712E-004	3.530E-003 0.000E+000	6.449E+000
400	2.312E-002 1.281E-003	5.696E-003 0.000E+000	1.361E+001
300	3.213E-002 1.629E-003	5.829E-002 0.000E+000	-.160E+002
200	3.632E-002 1.891E-003	1.690E-002 0.000E+000	1.027E+001
150	7.143E-002 2.781E-003	2.661E-001 0.000E+000	-.699E+002
100	6.537E-002 4.348E-003	1.351E-002 0.000E+000	1.193E+001
75	5.410E-002 3.565E-003	9.070E-002 0.000E+000	-.102E+002
60	6.494E-002 4.142E-003	1.875E-002 0.000E+000	1.115E+001
45	3.618E-002 1.982E-003	2.954E-002 0.000E+000	3.353E+000
30	2.014E-003 2.950E-004	3.441E-004 0.000E+000	5.659E+000
20	8.960E-006 7.639E-006	0.000E+000 0.000E+000	1.173E+000
10	0.000E+000 0.000E+000	0.000E+000 0.000E+000	0.000E+000

## Appendix D. Principle of Monte Carlo

### Introduction

The Monte Carlo Method uses random sampling to construct the solution of a physical or mathematical problem. This random sampling distinguishes this technique from numerical techniques. A complete treatment of the Monte Carlo method will not be attempted in this review, only the basic principles will be illustrated.

### Basic Principles

The Monte Carlo method depends on the use of "pseudorandom" numbers,  $\zeta$ , which are uniformly distributed on the interval (0,1), i.e.  $0 < \zeta < 1$ . The interested reader is referred to several references that discuss random number generation (Refs 12:27-31; 3).

The basic principle is based on modeling a physical phenomenon with the proper density function. If the phenomenon can be modeled as  $n$  independent, mutually exclusive events  $E_1, E_2, \dots, E_n$ , with probabilities  $p_1, p_2, \dots, p_n$ , respectively, and  $\sum_{i=1}^n p_i = 1$ , then for a discrete case  $E_i$  will be determined by the generation of a random number,  $\zeta$ , on the interval (0,1).  $P_i$  is the probability density function and is illustrated in Figure 1.

To find  $E_i$

$$P_{i-1} = p_1 + p_2 + \dots + p_{i-1} \leq \zeta < p_1 + p_2 + \dots + p_i = P_i$$

where  $P(x)$  is the cumulative probability distribution function for the discrete case. Figure 2 illustrates the distribution function.

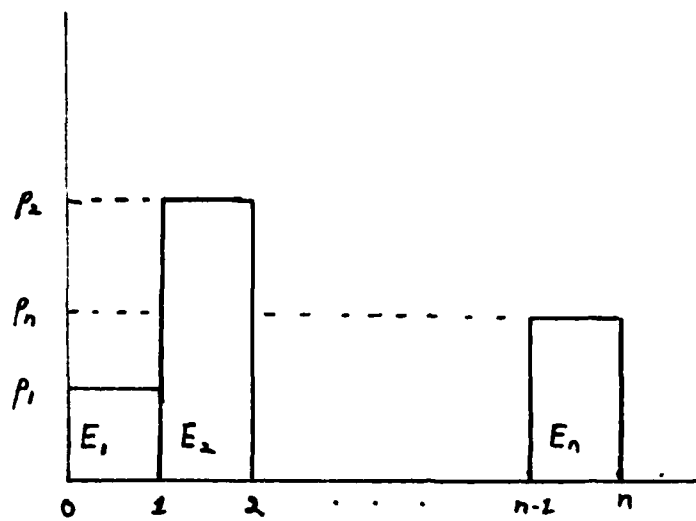


Figure 1. Density function

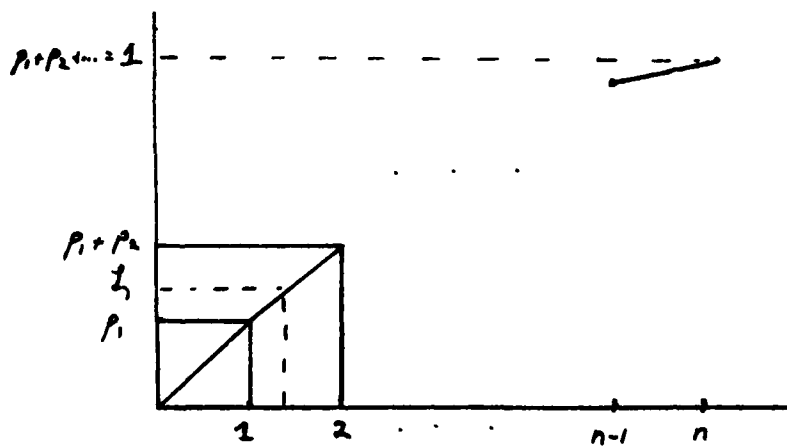


Figure 2. Cumulative probability function for the discrete case, event E, selected

Finding  $E_1$  for a continuous case may be approached in a similar fashion. If  $p(x)dx$  is the probability of  $x$  lying between  $x$  and  $x+dx$ , with  $a \leq x \leq b$ , and

$$\int_a^b p(x)dx = 1$$

then

$$\zeta = P(x) = \int_a^x p d$$

determines  $x$  uniquely as a function of  $\zeta$ ; if  $\zeta$  is uniformly distributed on  $0 \leq \zeta < 1$ , then  $x$  falls with frequency  $p(x)dx$  in the interval  $(x, x+dx)$ .

To illustrate this first method let us sample the distance to collision of a particle. The density function for this example is given by

$$p(l) = \Sigma_t e^{-\Sigma_t l} \quad (1)$$

where

$l$  = Distance to collision

$\Sigma_t$  = Total macroscopic cross-section of the medium.  $\Sigma_t$  is interpreted as the probability per unit length of a collision.

The probability of a first collision between  $l$  and  $l+dl$  along the photon's line of flight is given by

$$p(l)dl = \Sigma_t e^{-\Sigma_t l} dl \quad (2)$$

By applying the basic principle to equation (2),

$$\zeta = P(l) = \int_0^1 e^{-\Sigma_t s} \Sigma_t ds = 1 - e^{-\Sigma_t l} \quad (3)$$

solving for  $l$

$$l = \frac{1}{\sum_t} \ln(1-\zeta) \quad (4)$$

$l$  is then determined directly from  $\zeta$ .

Equation (4) is often seen simplified to

$$l = - \frac{1}{\sum_t} \ln \quad (5)$$

since  $1-\zeta$  is distributed in the same manner as  $\zeta$ .

This example illustrates the basic principles, but finding  $x$  becomes more difficult when the integral cannot be solved explicitly.

A simple method to overcome an implicit integral is to subdivide  $(a,b)$  into intervals, storing accurate values of  $P(X_i)=P_i$  for the end points of each subinterval

$$X_0 = a < x, \dots < x_n = b$$

Then using the discrete method for determining the interval  $(X_{i-1}, X_i)$  on which  $X$  falls. Interpolation is then used to find the final value of  $X$ . The resulting equation is given by

$$X = X_i - \frac{P(i)-\zeta}{P(i)-P(i-1)} (X_i - X_{i-1})$$

Often it is difficult or impossible to find a closed form for some density functions. An alternate method called the rejection technique may be used. The rejection technique is often used for sampling from a density function  $p(x)$ ,  $a < x < b$ , by employing two random numbers,  $\zeta$  and  $\eta$ . Figure 3 illustrates how the technique uses the random numbers to define points distributed on the rectangular area, bounded by the lines



$x=a, x=b, y=0$ , and  $y=1$ .

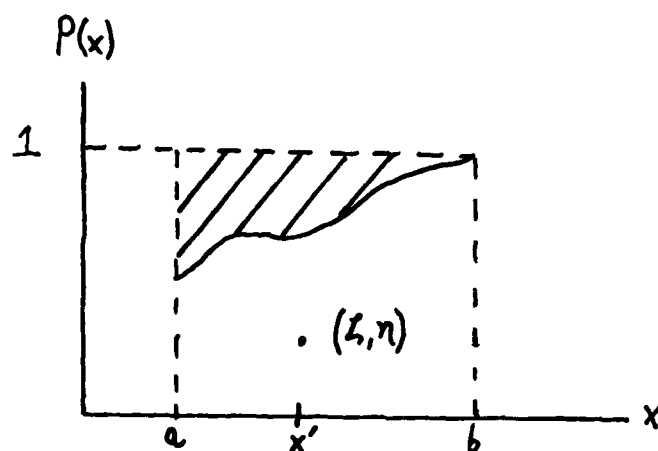


Figure 3. The rejection technique

This technique consists of "throwing" points  $(\zeta, \eta)$  uniformly into the bounded region and rejecting the points lying above the curve (in the shaded region). This technique is best used when the rejection area is only a small fraction of the total enclosed rectangle.

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### Vita

The author of this thesis, Lt. Alan W. Dooley, was born in Big Springs, Texas, on 26 October, 1959. He is the son of Mrs. Margaret Dooley and Lt. Colonel Floyd R. Dooley, Retired USAF, of Warner Robins, Georgia. Lt. Dooley graduated from Warner Robins High School before attending the University of Georgia for three years while studying for a chemistry degree. He then transferred to Georgia Institute of Technology where he received a Bachelor of Chemical Engineering in June 1982. The Air Force Institute of Technology was his first assignment.

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This report shows that gamma spectra identification by total flux correlation can be used to extend identification range over photo peak methods. Identification was based on two decision rules both employing cross-correlation coefficients. The largest coefficient (first decision rule) matched the unknown spectra with the correct source thirty-seven out of thirty-eight trials. The proposed likelihood function (second decision rule) had a success rate of thirty-five out of thirty-eight trials. These results were based on spectra generated by the transport code, Morse.

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